Separation of species of a binary fluid mixture confined in a channel in presence of a strong transverse magnetic field

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Abstract

Effects of a transverse magnetic field on the separation of a binary mixture of incompressible viscous thermally and electrically conducting fluids confined between two stationary parallel plates are examined. Both the plates are maintained at constant temperatures. It is assumed that one of the components, which is rarer and lighter, is present in the mixture in a very small quantity. The equations governing the motion, temperature and concentration in Cartesian coordinates are solved analytically. The solution obtained for concentration distribution is plotted against the width of the channel for various values of non-dimensional parameters. It is found that the effect of the transverse magnetic field is to separate the species of rarer and the lighter component by contributing its effect directly to the temperature gradient and the pressure gradient. The effects of increase in the values of the Hartmann number, magnetic Reynolds number, barodiffusion number, thermal diffusion number, electric field parameter and the product of the Prandtl number and Eckert number are to collect the rarer and lighter component near the upper plate and force the heavier component towards the lower plate. The problem discussed here derives its application in basic fluid dynamics separation processes to separate the rare component of the different isotopes of heavier molecules where the electromagnetic method of separation does not work.

Keywords: binary mixture, incompressible fluid, thermal diffusion, barodiffusion, rarer component, magnetic field.

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Separation processes of components of a binary fluid mixture wherein one of the components is present in extremely small proportion are of much interest due to their applications in science and technology. Besides environmental engineering applications, convection mass transfer alone constitutes the backbone of many operations in chemical industry. Separation of isotopes from their naturally occurring mixture is one such example. It is well known that only one part of heavy water, which is an isotope of water, is found in 25,000 parts of water in normal occurrence [1,2] but it is required for use as a *i*) moderator in nuclear reactions for slowing down the neutrons, *ii*) tracer compound for studying the mechanism of many chemical reaction and *iii*) heat transport medium, *i.e.*, a coolant in atomic power plants. Because of their small relative mass difference, isotopes of heavier molecules offer the greatest practical challenge in attempts to isolate the rarer component. Electromagnetic method of separation [3] works only at relatively higher values of concentrations.

Uranium is often grouped into a broader classification of contaminants particularly for drinking water, known as radionuclides. The most common radionuclides found in drinking water include uranium, radon and radium. Drinking water containing radionuclides can cause adverse health effects. As a result of non-biodegradable nature, the heavy metals including uranium accumulate in vital human organs and exert progressively growing toxic action [4]. Most notably, long-term ingestion of uranium and some other heavy metals may increase the risk of kidney damage, cancer and cardiovascular disease [5,6], whereas the experimental evidence suggests that the respiratory and reproductive systems are also affected by uranium exposure [7]. Hence, the public community water supplies must comply with the maximum contaminated limit (MCL) recommended by various National and International agencies like 15 [8], 30 [9], 9 ppb [7], etc. In many regions in India, the concentrations of radionuclides in drinking water are higher than the MCL [10]. The high concentration of radionuclides can be reduced to MCL by separating them from the water using the mechanism discussed in this work.

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In a binary fluid mixture the diffusion of individual species takes place by three mechanisms, namely ordinary diffusion, pressure diffusion (or barodiffusion) and thermal diffusion. The diffusion flux **i** of lighter and rarer component is given by [11], et al. as:

$$\mathbf{i} = -\rho D(\text{grad } c_1 + k_p \text{grad } p + k_T \text{grad } T) \quad (1)$$

where k_pD is the pressure diffusion coefficient and k_TD is the thermal diffusion coefficient. The ordinary diffusion contribution to the mass flux is seen to depend in a complicated way on the concentration gradients of the components present in the mixture. The barodiffusion indicates that there may be a net movement of the components in a mixture if there is a pressure gradient imposed on the system. An example of barodiffusion [12] is the process of diffusion in the binary mixture of different kinds of gases present in the atmosphere. By reasons of variation of forces of gravity with height thereby causing a density gradient, different constituents of the atmosphere tend to separate out. The pressure gradient created by the gravity as well as the rotation of the earth separates various components of air. The tendency for a mixture to separate under a pressure gradient is very small but use is made of this effect in centrifuge separations in which tremendous pressure gradient is established. Thermal diffusion describes the tendency for species to diffuse under the influence of a temperature gradient. In many practical problems dealing with flows in porous media one encounters with a multiple component electrically conducting fluids, *e.g.*, molten fluid in the earth's crust, crude oil in the petroleum. It is customary to consider one of the components as solvent and the other components as solute. It is shown [13] that if separation due to thermal diffusion occurs then it may even render an unstable system to stable one. This effect is also quite small, but devices can be arranged to produce very steep temperature gradients so that separations of mixtures are affected.

Sarma was perhaps the first who study the problem of barodiffusion in a binary mixture of incompressible viscous fluids set in motion due to an infinite disk rotation [14]. He obtained results on separation action in this configuration for small barodiffusion number taking the Schmidt number (*i.e.*, the ratio of viscous diffusion to mass diffusion) to be on the order of unity and including the effect of separation at the disk. He also discussed the effect of a temperature gradient on diffusion of a binary fluid mixture [14]. Many investigators [3, 14–39], analyzed the effects of barodiffusion and thermal diffusion on separation of a binary mixture in different geometries.

In many cases the fluid mixture is found to be electrically conducting. Therefore, in order to study the effect of magnetic field on separation, we have considered a binary mixture of incompressible viscous thermally and electrically conducting fluids confined between two stationary parallel plates in the presence of a strong transverse magnetic field. We have investigated the effect of a strong transverse magnetic field on the process of separation of the rarer component of a binary fluid mixture.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

We consider here the case when one of the components of the binary mixture of incompressible thermally and electrically conducting viscous fluids is present in small quantity, hence the density and viscosity of the mixture is independent of the distribution of the components. The concentration of the heavier and more abundant component is given by $c_2 = 1-c_1$. The flow problem of the binary mixture is identical to that of a single fluid but the velocity is to be understood as the mass average velocity $V = (\rho_1 V_1 + \rho_2 V_2)/\rho$ and the density $\rho = \rho_1 + \rho_2$, where the subscripts 1 and 2 denote the rarer and the more abundant component, respectively. The equation of continuity and the equation of motion of an incompressible fluid in steady case are respectively:

$$\nabla - V = 0 \quad (2)$$

and

$$\rho(\mathrm{V}\cdot\nabla)V = -\nabla \mathrm{p} + \rho\mathbf{F} + \mu\nabla^2\mathbf{V} + \mathbf{J}\times\mathbf{B} \quad (3)$$

In steady motion, the Maxwell equations are given by:

$$\text{curl } \mathbf{H} = 4\pi\mathbf{J} \quad (4)$$

$$\text{curl } \mathbf{E} = 0 \quad (5)$$

$$\text{div } \mathbf{H} = 0 \quad (6)$$

It is well known that for most of the fluids used in engineering applications collision frequency exceeds the cyclotron frequency for electrons. As the Hall current factor is the ratio of the cyclotron frequency to the collision frequency, the Hall current is very small and hence we have neglected it in our discussion. Consequently, Ohm's law is given by:

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V}\times\mathbf{B}) \quad (7)$$

with:

$$\mathbf{B} = \mu_e \cdot \mathbf{H} \quad (8)$$

where σ is the electric conductivity and μ_e is the magnetic permeability.

The energy equation in steady case is given by:

$$\rho c_p \mathbf{V}\cdot\nabla T = \kappa\nabla^2 \mathrm{T} + \mu\phi + \mathbf{J}^2/\sigma \quad (9)$$

where the last term $\mathbf{J}^2/\sigma$ represents heat due to electrical resistive dissipation.

The equation for species conservation of the first component is given by [40] as:

$$\rho(\mathbf{V}\cdot\nabla)c_1 = -\nabla\cdot\mathbf{i} \qquad (10)$$

where $\mathbf{i}$ is given by Eq. (1). The coefficients k_p and k_T may be determined from the thermodynamic properties alone. Reference [40] have given the explicit expression for the barodiffusion coefficient k_p as:

$$k_p = (m_2-m_1)(c_1/m_1 + c_2/m_2)c_1c_2/p_\infty \qquad (11)$$

Since $c_2 = 1-c_1$ and we have assumed c_1 to be very small, c_1^2 may be neglected and hence becomes:

$$k_p = (m_2-m_1)c_1/(m_2p_\infty) = Ac_1 \qquad (12)$$

where:

$$A = (m_2-m_1)/(m_2p_\infty) \qquad (13)$$

The expression for k_T has been suggested by [15] as:

$$k_T = s_Tc_1c_2 \qquad (14)$$

For small values of c_1, Eq. (14) becomes:

$$k_T = s_Tc_1 \qquad (15)$$

Substituting the expressions for $\mathbf{i}$ from Eq. (1), k_p from Eq. (12) and k_T from Eq. (15) in Eq. (10) we get the equation for c_1:

$$(\mathbf{V}\cdot\nabla)c_1 = D[\nabla^2c_1 + A\nabla\cdot(c_1\nabla P) + s_T\nabla\cdot(c_1\nabla T)] \qquad (16)$$

The boundary conditions for velocity are $\mathbf{V} = 0$ at solid surfaces since the surfaces are stationary. The boundary conditions for temperature are: the fluid temperatures at the surfaces are equal to a constant, which is the temperature of the surfaces of the cylinders. The boundary conditions for the concentration c_1 are different in different cases. At the surface of a body insoluble in the fluid mixture the total mass flux as well as the individual species flux normal to the surface should vanish [37]:

$$\rho c_1\mathbf{V}\cdot\mathbf{n} + \mathbf{i}\cdot\mathbf{n} = 0 \qquad (17)$$

Substituting the expression for $\mathbf{i}$ from Eq. (1) into Eq. (17), we get:

$$\rho c_1\mathbf{V}\cdot\mathbf{n} - \rho D(\nabla c_1\cdot\mathbf{n} + k_p\nabla p\cdot\mathbf{n} + k_T\nabla T\cdot\mathbf{n}] = 0 \qquad (18)$$

If, however, there is diffusion from a body that dissolves in the fluid, equilibrium is rapidly established near its surface, and the concentration in the fluid adjoining the body in this case is the saturation concentration c_0 (say); the diffusion out of this layer takes place more slowly than the process of solution. The boundary condition at such surface is, therefore:

$$c = c_0 \qquad (19)$$

FORMULATION OF THE PROBLEM

We consider here the steady flow of a binary mixture of thermally and electrically conducting viscous incompressible fluids by using the Cartesian coordinate system (x,y,z) as shown in Figure 1. The binary fluid mixture is confined between the walls of two infinite parallel plates separated by a distance 2h. The x-axis, which is parallel to the channel, is considered in the middle of the channel and the y-axis is perpendicular to the channel. The upper plate at $y = h$ is diffused to the fluid to establish the equilibrium near the surface, *i.e.*, c_0 and the lower plate at $y = -h$ is insoluble in the fluid, *i.e.*, impervious. Both plates are maintained at uniform constant temperatures, T_0. A strong uniform magnetic field of strength B_0 is applied in the transverse direction, and therefore the induced magnetic field b_x is developed in the x-direction. As we consider here, the flow is along the x-axis, so the flow depends only on y and the velocity vector is of the form $(u(y),0,0)$. The above geometry suggests that the magnetic field is of the form $(b_x,B_0,0)$ and the electric field is of the form $(0,0,E_z)$.

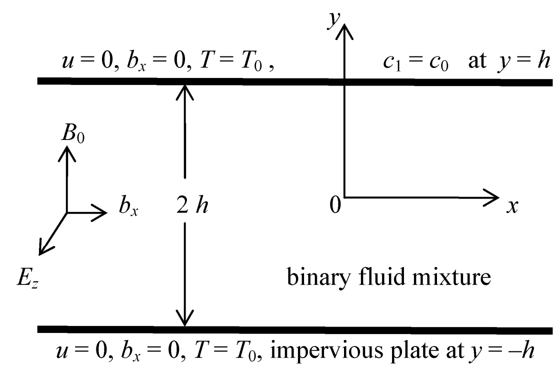

Figure 1. Physical geometry of the problem.

For the above-stated assumption, the governing Eqs. (3), (9) and (16) of the steady flow of a binary mixture of incompressible thermally and electrically conducting viscous fluids confined between two parallel plates in presence of a strong transverse magnetic field become:

$$-\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} - \sigma(E_z + uB_0)B_0 = 0 \qquad (20)$$

$$-\frac{\partial p}{\partial y} + \sigma(E_z + uB_0)B_0b_x = 0 \qquad (21)$$

$$B_0\frac{\partial u}{\partial y} + \frac{1}{\sigma\mu_e}\frac{\partial^2 b_x}{\partial y^2} = 0 \qquad (22)$$

$$\kappa\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma(E_z + uB_0)^2 = 0 \qquad (23)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial c_1}{\partial y}+Ac_1\frac{\partial p}{\partial y}+s_T c_1 r\frac{\partial T}{\partial y}\right)=0 \quad (24)$$

and the boundary conditions are:

$$\left.\begin{aligned}&u=0, b_x=0, T=T_0, c_1=c_0 \quad \text{at } y=h \text{ and}\\ &u=0, b_x=0, T=T_0, \frac{\partial c_1}{\partial y}+Ac_1\frac{\partial p}{\partial y}+s_T c_1\frac{\partial T}{\partial y}=0\\ &\text{at } y=-h\end{aligned}\right\} \quad (25)$$

To write the system of equations in a dimensionless form, we use the following variables transformations:

$$\left.\begin{aligned}&f(\eta)=\frac{u(y)}{-\frac{\partial u}{\partial x}\frac{h^2}{\mu}},\ b=\frac{b_x}{B_0},\ E=\frac{E_z}{\left(-\frac{\partial p}{\partial x}B_0\right)},\\ &g(\eta)=\frac{p(\eta)}{\left(-\frac{\partial p}{\partial x}\times h\right)}, \theta(\eta)=\frac{T-T_0}{T_0},\\ &c(\eta)=\frac{c_1(\eta)}{c_0},\ \text{where } \eta=\frac{y}{h}\end{aligned}\right\} \quad (26)$$

Using the transformations (26) in Eqs. (20)–(24), we get:

$$\frac{d^2 f}{d\eta^2}-M^2 f=M^2 R_e-1 \quad (27)$$

$$\frac{dg}{d\eta}=M^2(R_e+f)b \quad (28)$$

$$\frac{d^2 b}{d\eta^2}+R_m\frac{df}{d\eta}=0 \quad (29)$$

$$\frac{d^2\theta}{d\eta^2}+P_r E_c\left(\frac{df}{d\eta}\right)^2+M^2 P_r E_c\left(\frac{db}{d\eta}\right)^2=0 \quad (30)$$

$$\frac{d}{d\eta}\left(\frac{dc}{d\eta}+B_d c\frac{dg}{d\eta}+t_d c\frac{d\theta}{d\eta}\right)=0 \quad (31)$$

where

$$M=hB_0\sqrt{\sigma/\mu},\ R_e=E_z\Big/\left(-\frac{\partial p}{\partial x}\frac{h^2 B_0}{\mu}\right),$$

$$R_m=-\frac{\partial p}{\partial x}\sigma\mu_e h^3=R\times P_m,\ P_r=\frac{\mu c_p}{\kappa},$$

$$E_c=\left(-\frac{\partial p}{\partial x}\frac{h^2}{\mu}\right)^2\frac{1}{T_0 c_p},\ B_d=Ah\left(-\frac{\partial p}{\partial x}\right) \text{ and } t_d=S_T T_0,$$

and the boundary conditions (25) on velocity, magnetic field, temperature and concentration in terms of dimensionless quantities are:

$$\left.\begin{aligned}&f=0, b=0, \theta=0, c=1 \ \text{at } \eta=1 \text{ and}\\ &f=0, b=0, \theta=0, \frac{dc}{d\eta}+B_d c\frac{dg}{d\eta}+t_d c\frac{d\theta}{d\eta}=0 \text{ at } \eta=-1\end{aligned}\right\} \quad (32)$$

SOLUTION OF THE PROBLEM

The exact solutions of the Eqs. (27)–(31) subject to boundary conditions (30) are obtained and given by:

$$f(\eta)=\left(\frac{1}{M^2}-R_e\right)\left(1-\frac{\cosh M\eta}{\cosh M}\right) \quad (33)$$

$$\begin{aligned}\theta(\eta) &= -P_r E_c\left(\frac{1}{M^2}-R_e\right)^2\times\\ &\times[-\frac{1}{2}(1-\eta^2)+\frac{2}{M^2}\left(1-\frac{\cosh M\eta}{\cosh M}\right)-\\ &-\frac{2M^2(1-\eta^2)+(\cosh 2M-\cosh 2M\eta)}{4M^2(1+\cosh M)}+\\ &+M^2 R_m{}^2\{-\frac{1}{12}(1-\eta^4)-\\ &-\frac{(\cosh 2M-\cosh 2M\eta)-2M^2(1-\eta^2)}{4M^4(1+\cosh 2M)}+\\ &+\frac{2M(\sinh M-\eta\sinh M\eta)-4(\cosh M-\cosh M\eta)}{M^4\cosh M}\}]\end{aligned} \quad (34)$$

$$b=R_M\left(\frac{1}{M^2}-R_e\right)\left[\frac{1}{2}(1-\eta^2)-\frac{1}{M^2}\left(1-\frac{\cosh M\eta}{\cosh M}\right)\right] \quad (35)$$

$$\begin{aligned}g(\eta)-g(1) &= R_e R_m M^2\left(\frac{1}{M^2}-R_e\right)\times\\ &\times\left[-\frac{1}{3}+\frac{\eta}{2}-\frac{\eta^3}{6}+\frac{1}{M^2}\left(1-\eta-\frac{\sinh M-\sinh M\eta}{M\cosh M}\right)\right]+\\ &+R_m M^2\left(\frac{1}{M^2}-R_e\right)^2\times\\ &\times[\frac{1}{2}(1-\eta)^2\left(\eta-\frac{\sinh M\eta}{M\cosh M}\right)-\frac{1}{3}(1-\eta^3)+\\ &\frac{\cosh M-\eta\cosh M\eta}{M^2\cosh M}-\frac{\sinh M-\sinh M\eta}{M^3\cosh M}+\\ &+\frac{1}{M^2}\{1-\eta-\frac{2(\sinh M-\sinh M\eta)}{M\cosh M}+\\ &+\frac{2M(1-\eta)+\sinh 2M-\sinh 2M\eta}{2M(1+\cosh 2M)}\}]\end{aligned} \quad (36)$$

and

$$c(\eta)=\exp\{-B_d(g(\eta)-g(1))-t_d\theta(\eta)\} \quad (37)$$

RESULTS

In the absence of a magnetic field, Eq. (37) produces a singular solution. So, putting $B_0 = 0$ in Eqs. (27)–(31) and solving under the boundary conditions (32), we get:

$$c(\eta) = \exp\left\{-\frac{t_d p_r E_c}{12}\left(1-\eta^4\right)\right\} \tag{38}$$

If we put t_d and $B_d = 0$ in expressions (37) and (38) for concentration distribution of the first component of the binary fluid mixture we get $c(\eta) = 1$ for all values of η. From this we can conclude that the separation of species ceases to take place if we neglect the combine effect of the thermal diffusion number and barodiffusion coefficient. Our results are found to be in good agreement with the results of the researchers [3, 4–15, 18,20–26, 28–31, 33–39,41,42].

Figures 2– 4 and 6 reveal that the concentration of the rarer and lighter species decreases near the lower plate with increasing Hartmann number, *i.e.*, the strength of the magnetic field M, electric field parameter, magnetic Reynolds number and barodiffusion number. This suggests that the species of rarer and lighter component thrown away towards the upper plate by increasing the strength of the magnetic field consequently the electric field, strength of magnetic Reynolds number, *i.e.*, the Reynolds number as well as the magnetic Prandtl number, and strength of barodiffusion number representing the pressure gradient in the system. For M = 19 nearly all the concentration of the rarer and lighter species of the binary fluid mixture moves towards the upper plate and similar phenomenon happens for a certain value (Table 1) in case of the electric field parameter, magnetic Reynolds number and barodiffusion number. Hence, the species of the rarer and lighter component of the binary fluid mixture can be separated by increasing the values of the Hartmann number, M, electric field parameter, R_e, magnetic Reynolds number, R_m, and barodiffusion number, B_d.

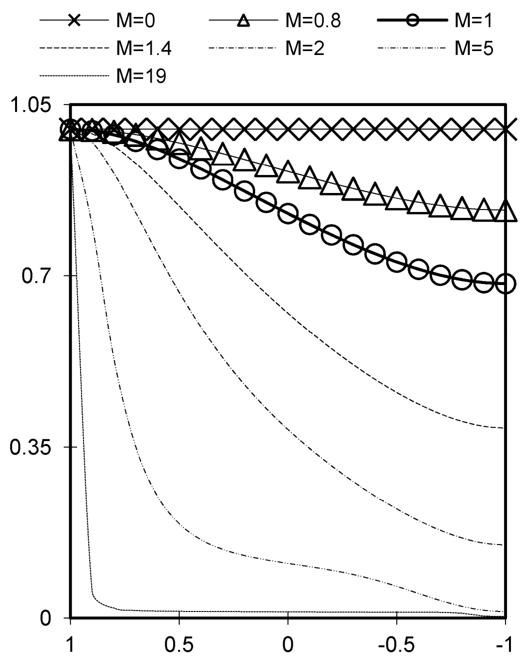

Figure 2. The graphs of the concentration profile c(η) against the width of the channel taking R_e = 10, R_m = 30, PrE_c = 0.0001, B_d = 0.001, t_d = 0.001 for various values of the Hartmann number.

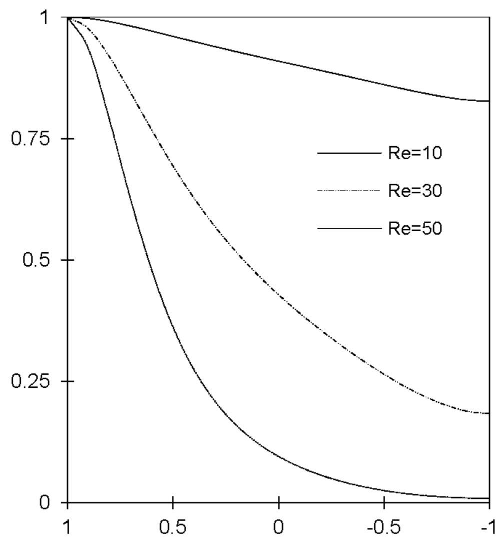

Figure 3. The graphs of the concentration profile c(η) against the width of the channel taking M = 2, R_m = 30, $PrE_c = 1.0\times10^{-5}$, B_d = 0.0001, t_d = 0.0001 for various values of the electric field parameter.

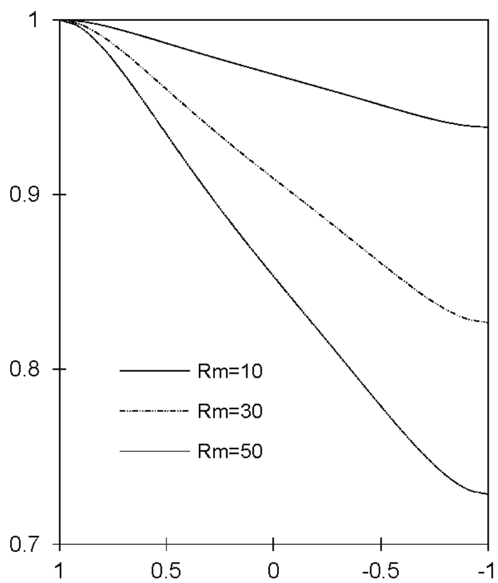

Figure 4. The graphs of the concentration profile c(η) against the width of the channel taking M = 2, R_e = 10, P_rE_c = 0.0001, t_d = 0.0001, B_d = 0.0001 for the various values of the magnetic Reynolds number.

Figures 5 and 7 reveal that the concentration of the rarer and lighter component of the binary fluid mixture decreases in between the channel by increasing the strength of Prandtl number, Eckert number and the thermal diffusion number but the concentration of the rarer and lighter species remains constant for any value of the above parameter. Thus, the species of the rarer and lighter element can be separated by changing the values of the parameters P_r, E_c and t_d conveniently. Table 2 shows that the concentration profiles of the rarer and lighter component of the binary mixture are nearly overlapping for the values of parameters M = 0 and R_e = 0 but increments 0.8 and 2 in the value of M shows the same trends for the increments 3 and 10 respectively in the values of R_e (Table 2). Therefore, it can be concluded that the influence of magnetic field is more than that of the electric field on separation of the binary fluid mixture.

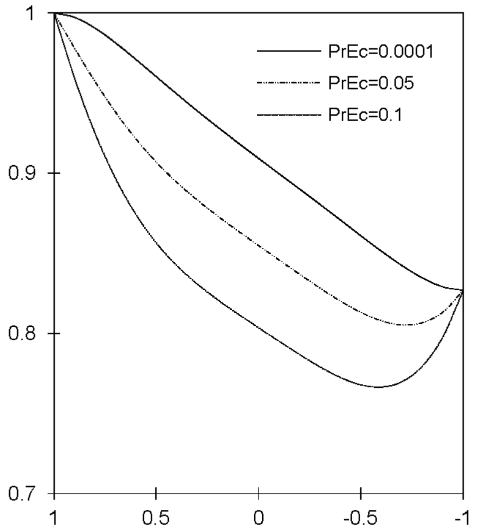

Figure 5. The graphs of the concentration profile c(η) against the width of the channel taking M = 2, R_e = 10, R_m = 30, B_d = 0.0001, t_d = 0.000 for the various values of the product of Prandtl's number and Eckert's number.

From the discussion above, we can conclude that the magnetic field, electric field parameter, magnetic Reynolds number, product of Prandtl number and Eckert number, barodiffusion number and the thermal diffusion number affect the process of separation in the binary fluid mixture. Hence, separation of the rarer and lighter species of the binary fluid mixture can be done by taking the suitable values of the parameters. From Figures 2–7 it can be concluded that the barodiffusion number, B_d, representing the pressure gradient is the most effective parameter in the process of separation.

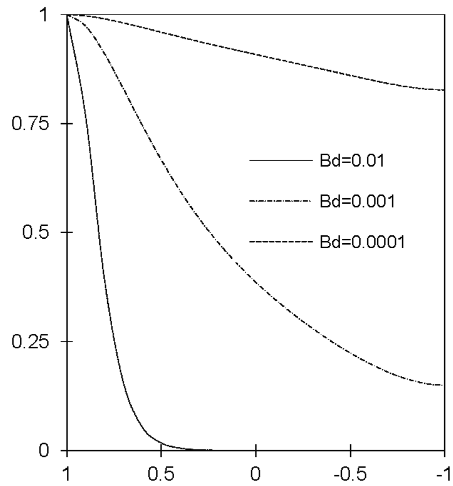

Figure 6. The graphs of the concentration profile c(η) against the width of the channel taking M = 2, P_rE_c = 0.0001, t_d = 0.001, R_e = 10, R_m = 30 for the various values of the rotational Reynolds number.

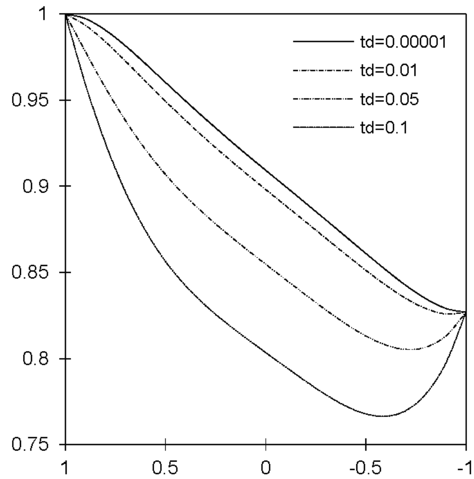

Figure 7. The graphs of the concentration profile c(η) against the width of the channel taking M = 2, P_rE_c = 0.0001, B_d = 0.0001, R_e = 10, R_m = 30 for the various values of the rotational Reynolds number.

Table 1. Values of parameters M, R_e, R_m and B_d for which c(η) values confirm that rarer and lighter component is almost collected near the upper plate of the channel

η	R_e = 10, R_m = 30, P_rE_c = 0.0001, B_d = 0.001, t_d = 0.001		M = 2, R_m = 30, P_rE_c = 0.00001, B_d = 0.0001, t_d = 0.0001		M = 2, R_e = 10, P_rE_c = 0.0001, B_d = 0.0001, t_d = 0.0001		M = 2, R_e = 10, R_m = 30, P_rE_c = 0.0001, t_d = 0.001	
	M = 19	M = 21	R_e = 200	R_e = 250	R_m = 6500	R_m = 7500	B_d = 0.05	B_d = 0.1
1	1	1	1	1	1	1	1	1
0.9	0.055275	9.8E–05	0.335392	0.181349	0.092104	0.046312	0.263046	0.069196
0.8	0.020553	3.52E–05	0.021672	0.002508	0.005319	0.001347	0.009061	8.21E–05

Table 1. Continued

η	R_e = 10, R_m = 30, P_rE_c = 0.0001, B_d = 0.001, t_d = 0.001		M = 2, R_m = 30, P_rE_c = 0.00001, B_d = 0.0001, t_d = 0.0001		M = 2, R_e = 10, P_rE_c = 0.0001, B_d = 0.0001, t_d = 0.0001		M = 2, R_e = 10, R_m = 30, P_rE_c = 0.0001, t_d = 0.001	
	M = 19	M = 21	R_e = 200	R_e = 250	R_m = 6500	R_m = 7500	B_d = 0.05	B_d = 0.1
0.7	0.015621	2.79E–05	0.000518	7.34E–06	0.00026	3.45E–05	8.93E–05	7.97E–09
0.6	0.014374	2.59E–05	7.36E–06	9.52E–09	1.3E–05	9.64E–07	4.44E–07	1.97E–13
0.5	0.013856	2.5E–05	8.54E–08	9.01E–12	7.4E–07	3.26E–08	1.64E–09	2.7E–18
0.4	0.013538	2.44E–05	9.87E–10	8.48E–15	4.98E–08	1.39E–09	5.77E–12	3.33E–23
0.3	0.013296	2.39E–05	1.26E–11	9.31E–18	3.94E–09	7.26E–11	2.19E–14	4.78E–28
0.2	0.013088	2.35E–05	1.83E–13	1.25E–20	3.54E–10	4.47E–12	9.39E–17	8.81E–33
0.1	0.012894	2.32E–05	2.96E–15	2E–23	3.47E–11	3.06E–13	4.51E–19	2.04E–37
0	0.012703	2.29E–05	5.12E–17	3.55E–26	3.53E–12	2.19E–14	2.32E–21	5.37E–42
–0.1	0.012516	2.25E–05	8.86E–19	6.28E–29	3.6E–13	1.57E–15	1.19E–23	1.42E–46
–0.2	0.01234	2.22E–05	1.44E–20	1.01E–31	3.6E–14	1.1E–16	5.72E–26	3.27E–51
–0.3	0.01219	2.19E–05	2.09E–22	1.35E–34	3.55E–15	7.69E–18	2.46E–28	6.03E–56
–0.4	0.012086	2.18E–05	2.66E–24	1.48E–37	3.58E–16	5.57E–19	9.31E–31	8.67E–61
–0.5	0.012046	2.18E–05	3.07E–26	1.4E–40	3.97E–17	4.6E–20	3.27E–33	1.07E–65
–0.6	0.012042	2.19E–05	3.57E–28	1.33E–43	5.45E–18	5.03E–21	1.21E–35	1.46E–70
–0.7	0.011764	2.19E–05	5.08E–30	1.72E–46	1.1E–18	8.97E–22	6.01E–38	3.62E–75
–0.8	0.009811	1.95E–05	1.22E–31	5.05E–49	4.1E–19	3.45E–22	5.93E–40	3.51E–79
–0.9	0.004399	1.03E–05	7.88E–33	7.02E–51	3.83E–19	4.08E–22	2.04E–41	4.17E–82
–1	0.003274	0.003127	2.65E–33	1.28E–51	1.31E–18	2.32E–21	5.37E–42	2.88E–83

Table 2. Values of c(η) to estimate the influence of electric force (represented by parameter R_e) and the Lorentz force (represented by parameter M)

η	R_e = 10, R_m = 30, P_rE_c = 0.0001, B_d = 0.001, t_d = 0.001			M = 2, R_m = 30, P_rE_c = 0.0001, B_d = 0.001, t_d = 0.001		
	M = 0	M = 0.8	M = 2	R_e = 0	R_e = 3	R_e = 10
1	1	1	1	1	1	1
0.9	0.999999995	0.998661558	0.973269886	1.00000205	0.997694319	0.973269886
0.8	0.999999993	0.99484847	0.909594075	1.00001648	0.9918883	0.909594075
0.7	0.999999992	0.988871252	0.829135299	1.000051677	0.98387891	0.829135299
0.6	0.999999992	0.981058009	0.74558175	1.000112247	0.974621073	0.74558175
0.5	0.999999992	0.97173892	0.666530253	1.000199644	0.964772453	0.666530253
0.4	0.999999992	0.961235549	0.59525185	1.000312791	0.954751581	0.59525185
0.3	0.999999992	0.949854278	0.53243248	1.000448655	0.944795406	0.53243248
0.2	0.999999993	0.937883048	0.477436669	1.000602741	0.935010173	0.477436669
0.1	0.999999995	0.925590689	0.429095977	1.000769526	0.925414063	0.429095977
0	1	0.913228087	0.386156243	1.000942835	0.915972378	0.386156243
-0.1	1	0.901030633	0.347513635	1.001116175	0.906627052	0.347513635
-0.2	1	0.889221405	0.312329221	1.001283049	0.897322586	0.312329221
-0.3	1	0.878014701	0.280073837	1.001437277	0.888030442	0.280073837
-0.4	1	0.867619571	0.250529625	1.001573328	0.878773701	0.250529625
-0.5	1	0.858243082	0.223762133	1.001686701	0.869653504	0.223762133
-0.6	1	0.850093059	0.200075131	1.00177436	0.860878508	0.200075131
-0.7	1	0.84338004	0.179966803	1.001835231	0.852798265	0.179966803
-0.8	1	0.83831813	0.164119052	1.001870778	0.84594127	0.164119052
-0.9	1	0.835124308	0.15347294	1.001885627	0.841058188	0.15347294
-1	1	0.834015542	0.14948599	1.001888189	0.83917053	0.14948599

CONCLUSIONS

The problem of mass transfer due to the flow of an electrically and thermally conducting viscous incompressible binary fluid mixture between two horizontal parallel plates under the influence of a strong magnetic field acting in the transverse direction, has been investigated under the assumption that one of the components, which is rarer and lighter, is present in the mixture in a very small quantity. Analytical solutions of the governing equations have been obtained in closed form. Different analytic expressions are obtained for non-dimensional velocity, induced magnetic field, temperature and concentration profile in the presence of a strong transverse magnetic field. The specific conclusions derived from this study can be listed as follows:

- The combined effect of the thermal diffusion number and the barodiffusion number is to separate the species of the binary fluid mixture, *i.e.*, in the absence of the barodiffusion number and the thermal diffusion number separation of species ceases to take place.
- The magnetic field alone cannot affect the separation of the species in the absence of both the pressure gradient and the temperature gradient.
- At certain values of the Hartmann number, electric field parameter, magnetic Reynolds number and barodiffusion number, almost all the rater and lighter component of binary fluid mixture is brought towards the upper plate of the channel (Table 1).
- The species of the rarer and lighter component of the binary mixture is collected towards the upper plate with the increase in the values of parameters Hartmann number, electric field parameter, magnetic Reynolds number and barodiffusion number.
- The product of Prandtl number and the Eckert number and the thermal diffusion number affects the process of separation only in the channel keeping the concentration unaffected in the interface of the plates and the fluid.
- The influence of the magnetic field is greater than that of the electric field.

Taking into account the conclusions derived in this paper, gas-separating instruments can be installed, as an engineering application, in big cities where harmful gases are present in very small quantities that can be sucked after separating them and thus pollutants can be removed.

List of symbols

B Magnetic inductance vector
B_0 Uniform applied magnetic field
B_d Barodiffusion number
b_x Induced magnetic field along the plate
c Concentration
c_1 Concentration of the first component of the binary mixture
c_2 Concentration of the second component of the binary mixture
c_p Specific heat at constant pressure
c_0 Concentration of lighter and rarer component of the binary fluid mixture at upper plate
D Diffusion coefficient
E Electric field vector
E_c Eckert number
E_z Component of electric field along z-direction
F Body force per unit mass
H Magnetic field vector
h Half width of the channel
i Diffusion flux density vector
J Current density vector
k_p Barodiffusion ratio
k_T Thermal diffusion ratio
M Hartmann number
m_1 Mass of first kind of the particle
m_2 Mass of second kind of the particle
n Unit vector drawn perpendicular to the plates
p Pressure
P_m Magnetic Prandtl number
p_∞ Working pressure of the medium
P_r Prandtl number
R Reynolds number
R_e Electric field parameter
R_m Magnetic Reynolds number
S_T Soret coefficient
T Temperature, T_0, temperature of the plates
t_d Soret number
u Velocity along x-direction
V Average velocity
$\mathbf{V_1}$ Velocity of rarer and lighter component
$\mathbf{V_2}$ Velocity of more abundant component
x Co-ordinate measuring the distance parallel to the plate
y Co-ordinate measuring the distance perpendicular to the plate
z Co-ordinate measuring the distance perpendicular to both *x*-axis and *y*-axis.

Greek symbols

ϕ Heat due to viscous dissipation
η Non-dimensional variable measuring the distance perpendicular to the plate
η_m Coefficient of magnetic viscosity or magnetic diffusivity
κ Thermal conductivity
μ Coefficient of viscosity
μ_e Magnetic permeability
ν Coefficient of kinematic viscosity
θ Non-dimensional temperature
ρ Density of binary fluid mixture
ρ_1 Density of the first component of the binary mixture

ρ_2 Density of the second component of the binary mixture

σ Electrical conductivity.

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IZVOD

SEPARACIJA BINARNE SMEŠE FLUIDA OMEĐENE KANALOM U PRISUSTVU JAKOG TRANSVERZALNOG MAGNETNOG POLJA

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(Naučni rad)

U ovom radu, ispitivano je dejstvo transverzalnog magnetnog polja na separaciju binarne smeše viskoznih toplotno i elektroprovodnih fluida omeđenih sa dve stacionarne paralelne ploče. Temperatura obe ploče je bila konstantna. Pretpostavljeno je da je jedna od komponenti, koja je ređa i lakša, prisutna u smeši u veoma maloj količini. Rešenja jednačina koje opisuju kretanje, temperaturu i koncentraciju u Dekartovim kordinatama su dobijena analitičkim putem. Rešenje dobijeno za raspodelu koncentracije je prikazano u zavisnosti od širine kanala za različite vrednosti bezdimenzionih parametara. Nađeno je da trasverzalno magnetno polje utiče na separaciju ređe i lakše komponente tako što neposredno utiče na gradijent temperature i gradijent pritiska. Povećanje vrednosti Hartmanovog broja, Rejnoldsovog broja, barodifuzionog broja, toplotnog difuzionog broja, parametra električnog polja i proizvoda Prantlovog i Ekertovog broja utiče na sakupljanje ređe i lakše komponente bliže gornjoj ploči i odbacivanje teže komponentu prema donjoj ploči. Problem razmatran u ovom radu podstaknut je onim separacionim procesima u kojima elektromagnetne metode za razdvajanje retkih komponenti različitih izotopa težih molekula nisu moguće.

Ključne reči: Binarna smeša • Nestišljivi fluid • Toplotna difuzija • Barodifuzija • Ređa komponenta • Magnetno polje