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NEW SIMPLE ALGEBRAIC ROOT LOCUS METHOD FOR DESIGN OF FEEDBACK CONTROL SYSTEMS

New concept of algebraic characteristic equation decomposition method is presented to simplify the design of closed-loop systems for practical applications. The method consists of two decompositions. The first one, decomposition of the characteristic equation into two lower order equations, was performed in order to simplify the analysis and design of closed loop systems. The second is the decomposition of Laplace variable, s, into two variables, damping coefficient, ζ, and natural frequency, ω_n. Those two decompositions reduce the design of any order feedback systems to setting of two complex dominant poles in the desired position. In the paper, we derived explicit equations for six cases: first, second and third order system with P and PI. We got the analytical solutions for the case of fourth and fifth order characteristic equations with the P and PI controller; one may obtain a complete analytical solution of controller gain as a function of the desired damping coefficient. The complete derivation is given for the third order equation with P and PI controller. We can extend the number of specified poles to the highest order of the characteristic equation working in a similar way, so we can specify the position of each pole. The concept is similar to the root locus but root locus is implicit, which makes it more complicated and this is simpler explicit root locus. Standard procedures, root locus and Bode diagrams or Nichol Charts, are neither algebraic nor explicit. We basically change controller parameters and observe the change of some function until we get the desired specifications. The derived method has three important advantage over the standard procedures. It is general, algebraic and explicit. Those are the best poles design results possible; it is not possible to get better controller design results.

It has been stated many times that the chemical process control theory is inadequate, resulting in poor connection with application. The good theory must be useful not only to process control engineers for design, but should be also developed to accommodate the need and skills of the potential users in application. You cannot expect users to understand complicated theories, so the best control theory should be as simple as possible.

Laplace transform simplifies the linear differential equations into algebraic polynomial equations in the Laplace transform variable, s. Algebraization is very important simplification which we use for controller design. A root locus curve is a plot of the roots of the closed-loop characteristic equation as a function of the controller gain. As the gain rise the root locus for third order system changes so that two critical roots move toward the limit of stability of critical axe and one non--critical move in less critical region. We design the P controller by choosing P so that the relative stability of the closed-loop system to have a damping coefficient of about 0.3.

The procedure includes graphing polynomial roots, drawing lines of constant damping coefficient and crossing this two lines and estimating the value of P controller. This procedure is not simple, it is iterative and it is not analytic. This is the standard procedure used in many various new textboxes [1–9].

In this paper, we will develop a simpler concept to get the explicit values of P and PI controller that will locate the closed loop poles in the proper position with the required damping coefficient (relative stability) value in one step without graphing the root locus.

In our paper, the new obtained results are more complicated then the root locus analysis, but analytical results may be obtained for the higher order transfer functions (n = 3, 4, 5).

THE METHOD DEVELOPMENT

Roffel and Rijnsdorp [10] used the polynomial decomposition method for the calculations of frequency stability limits with the domain decomposition for the value of $\zeta = 0$. They divided the system closed loop characteristic equation with the value $(s^2 + \omega_u^2)$ to get the stability limits. Their detailed calculation is presented in the case of third order system with P controller. But for the controller design we need relative stability, not the stability limits.

To solve the problem of relative stability, Mitrović [11] proposed a change of variable s of the form $s = -\zeta\omega_n\sqrt{1-\zeta^2}$, into the characteristic equation, and obtained complex equation with two variables, ζ and ω_n. In the book of Thaler [12], Chapter 10 is devoted to this method.

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Mitrović's method is different and much more complicated then method of Roffel and Rijnsdorp. In this paper, we will show how to improve the quantitative possibilities of the standard methods of complex variables combining the two above methods.

On the first polynomial decomposition level, we will decompose high order polynomial into two lower level polynomials as Roffel and Rijnsdorp [10] did. The second level of decomposition is the domain decomposition. The Laplace complex domain, s, is decomposed into two real subdomains ζ and ω_n, like Mitrovic [11] did. The double decomposition results in decomposition of the closed loop characteristic equation into three equations in real domains (ζ, ω_n). During decompositions, the complex number calculations are not used.

The obtained equations are in such a form that all standard mathematical methods for the real function of real variable can be used without any problems. The physical meaning is the usual. The complex equations and complex domain completely disappear.

It is easier to solve many lower order equations than a few higher order one. Hence, using decomposition, one may analytically solve more problems. For example, it is possible to set more poles into the desired positions, without solving high order polynomials. This analytical method is global in the sense that it is easier to analyze the effect of changes of all parameters included. Consequently, the method can be used to compare control loops with different transfer functions and closed loop structures. This kind of analysis is not possible to conduct with classical local design structure.

The method provides a clear picture of absolute stability and response speed and some picture of relative stability. Both parameters in the PI controller provide limits of parameter values and one may estimate the effect of parameter design on process response.

One can argue that analytical methods cannot be competed with today's high computational power available to engineers. But analytical methods are the engineering way to understand and solve problems and to control the computer calculations, and one must also strengthen the leading role of engineer instead of searching for algorithms [13]. The point is that we need both; human and computer power, and those powers are not comparable, they are interactive and compatible.

The new derived method has three important advantages over the standard procedures: It is general, algebraic and explicit, which is significant advantage over the standard methods. Standard methods are not general, not algebraic and not explicit.

When designing the P controller with standard specific frequency or root locus method, we have to draw different graph for every single specific case, so the methods requires to repeat the calculation procedure for every particular design. The results presented in this paper are better:

1. general means that we have derived general algebraic formulas for all first, second and third order system with P and PI controller. Using those formulas we can solve all first, second and third order problems;

2. algebraic and explicit means that if we insert the specific data for the process and closed loop specification we get the controller design explicitly in one step.

The equation derived in this paper solved the case of first, second and third order process with P and PI controller explicitly for every possible case. I derived the equations up to the 5th order. There is no reason that higher order equation cannot be solved manually or by computer program. All the equations have a clear physical meaning and all the parameters of the characteristic equations are connected analytically with fundamental parameters ζ and ω_n. We will now apply new method systematically on first, second and third order system. In the case of third order system with P controller, we will compare the Roffel and Rijnsdorp method with the new one.

THE APPLICATION OF THE NEW METHOD FOR THE ANALYSIS AND SYNTHESIS OF FIRST ORDER PROCESS AND *P* CONTROLLER

Consider the given first order open-loop transfer function:

$$g(s) = \frac{1}{a_1 s + a_0} \tag{1}$$

The closed-loop characteristic equation with P controller is:

$$a_1 s + a_0 + K = 0 \tag{2}$$

where K is the controller gain. If we specify the closed loop pole position with ($a_{1c}s + a_{0c}$), where index 1c means specified closed loop parameters, then we can find the appropriate K by dividing the closed loop characteristic equation with the specified closed loop polynomial:

$$(a_1 s + a_0 + K) : (a_{1c} s + a_{0c}) = \frac{a_1}{a_{1c}} \tag{3}$$

$$[-a_1 s - a_{0c}\frac{a_1}{a_{1c}}]$$

$$a_0 + K - a_{0c}\frac{a_1}{a_{1c}} \tag{4}$$

In order to make that the above denominator divide evenly into the numerator, the remainder (4) must be zero and:

$$K = a_{0c}\frac{a_1}{a_{1c}} - a_0 \tag{4a}$$

So here is the result, if we want position the closed loop pole into the desired position, K has to have the value given by Eq. (4a).

THE APPLICATION OF THE NEW METHOD FOR THE ANALYSIS OF FIRST ORDER PROCESS AND PI CONTROLLER

Consider the given first order open-loop transfer function (1).

The closed-loop characteristic equation with PI controller is divided with the specified second order term:

$$[a_1 s^2 + (a_0 + K)s + \frac{K}{T_I}] : (s^2 + 2\zeta\omega_n s + \omega_n^2) = a_1 \quad (5)$$

$$[-a_1 s^2 - 2a_1\zeta\omega_n s - a_1\omega_n^2]$$

$$(a_0 + K - 2a_1\zeta\omega_n)s + \frac{K}{T_I} - a_1\omega_n^2 \quad (6)$$

To make remainder (6) equal zero two equations must be satisfied:

$$K = 2a_1\zeta\omega_n - a_0 \quad (7)$$

and

$$T_I = \frac{2a_1\zeta\omega_n - a_0}{a_1\omega_n^2} \quad (8)$$

And these are the values of controller K and T_I that will move the two poles from the given open loop values to the specified complex values.

THE APPLICATION OF THE NEW METHOD FOR THE ANALYSIS AND SYNTHESIS OF SECOND ORDER PROCESS AND P CONTROLLER

Consider the second order open-loop transfer function:

$$g(s) = \frac{1}{a_2 s^2 + a_1 s + a_0} \quad (9)$$

The closed-loop characteristic equation with P controller is:

$$a_2 s^2 + a_1 s + a_0 + K = 0 \quad (10)$$

where K is the controller gain.

If we want to design the system with a desired closed-loop damping coefficient of ζ and ω_n, where ω_n is the undamped natural frequency, then it is proposed that Eq. (9) is divided by polynomial:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 \quad (11)$$

The result is:

$$(a_2 s^2 + a_1 s + a_0 + K) : (s^2 + 2\zeta\omega_n s + \omega_n^2) = a_2 \quad (12)$$

$$[-a_2 s^2 - 2a_2\zeta\omega_n s - a_2\omega_n^2]$$

$$(a_1 - 2a_2\zeta\omega_n)s + a_0 + K - a_2\omega_n^2$$

In order to make that the above denominator divide evenly into the numerator, the remainder (last row) must be zero and two Equations, (13) and (14), must be satisfied:

$$a_1 - 2a_2\zeta\omega_n = 0 \quad (13)$$

$$a_0 + K - a_2\zeta\omega_n^2 = 0 \quad (14)$$

Solving Eqs. (9)–(13) and (10)–(14) results in:

$$K = \frac{a_1\omega_n}{2\zeta} - a_0 \quad (15)$$

The problem is solved forever for all possible process parameters and closed-loop specifications ζ and ω_n. We can also analyze how the result varies with parameters.

THE APPLICATION OF THE NEW METHOD FOR THE ANALYSIS OF SECOND ORDER PROCESS AND *PI* CONTROLLER

Consider the second order open-loop transfer function given by Eq. (9).

The closed-loop characteristic equation with PI controller is divided with the specified second order term:

$$\begin{aligned}&[a_3 s^3 + a_1 s^2 + (a_0 + K)s + \frac{K}{T_I}] : (s^2 + 2\zeta\omega_n s + \omega_n) = \\ &= a_3 s^2 + (a_2 - 2a_3\zeta\omega_n)s + a_1 - a_3\omega_n^2 - \\ &\quad - 2\zeta\omega_n(a_2 - 2a_3\zeta\omega_n)\end{aligned} \quad (16)$$

To make remainder equal zero two equations must be satisfied:

$$K = a_2\omega_n^2 + 2\zeta\omega_n(a_1 - 2a_2\zeta\omega_n) - a_0 \quad (17)$$

and

$$T_I = \frac{a_2\omega_n^2 + 2\zeta\omega_n(a_1 - 2a_2\zeta\omega_n) - a_0}{\omega_n^2(a_1 - 2\zeta\omega_n a_2)} \quad (18)$$

And these are the values of controller K and T_I that will move the two poles from the given open loop values to the specified complex values.

THE APPLICATION OF THE NEW METHOD FOR THE ANALYSIS AND SYNTHESIS OF THIRD ORDER PROCESS AND P CONTROLLER

Consider the third order open-loop transfer function:

$$g(s) = \frac{1}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{19}$$

The closed-loop characteristic equation with P controller is:

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0 + K = 0 \tag{20}$$

One can obtain the parameters for the limits of stability dividing Eq. (19) by the expression $(s^2 + \omega_u^2)$, as Roffel and Rijnsdorp [10] did, where ω_u is the ultimate frequency. The quotient is $(a_3 s + a_2)$ and the remainder is $((a_1 - a_3\omega_u^2)s + a_0 + K_u - a_2\omega_u^2)$, where K_u is the ultimate gain. One wants the denominator to divide evenly into the numerator, so that the remainder equals to zero and one gets two equations with a solutions:

$$\omega_u^2 = \frac{a_1}{a_3}, \quad K_u = a_2 \frac{a_1}{a_3} - a_0 \tag{21}$$

If Relation (21) is satisfied then the polynomial of Eq. (20) can be decomposed into the product of two lower order polynomials:

$$(s^2 + \omega_u^2)(\frac{a_1}{\omega_u^2} s + \frac{a_3}{a_1} a_0 + K_u) = 0 \tag{22}$$

The first term gives the desired position of system poles, at the limit of stability, and the second term gives the position of the third pole. At the same time, the third order characteristic equation is solved analytically by decomposition.

Analyzing both methods one get the idea to combine the method of decomposition with the method of Mitrović in the following way.

If one wants to design the system with a desired closed-loop damping coefficient of ζ, it is proposed that Eq. (20) is divided by polynomial Eq. (23):

$$s^2 + 2\zeta\omega_n s + \omega_n^2 \tag{23}$$

The result is:

$$\begin{array}{l} (a_3 s^3 + a_2 s^2 + a_1 s + a_0 + K) : (s^2 + 2\zeta\omega_n s + \omega_n^2) = a_3 s + a_2 - 2a_3\zeta\omega_n \\ a_3 s^3 + 2a_3\zeta\omega_n s^2 + a_3\,\omega_n^2\, s \\ \text{----------} \\ (a_2 - 2a_3\zeta\omega_n)s^2 + (a_1 - a_3\omega_n^2)s + a_0 + K \\ \dfrac{(a_2 - 2a_3\zeta\omega_n)s^2 + 2\zeta\omega_n s(a_2 - 2a_3\zeta\omega_n))s + \omega_n^2(a_2 - 2a_3\zeta\omega_n)}{(a_1 - a_3\omega_n^2 - 2\zeta\omega_n s(a_2 - 2a_3\zeta\omega_n))s + a_0 + K - \omega_n^2(a_2 - 2a_3\zeta\omega_n)} \end{array} \tag{24}$$

In order to make that the above denominator divide evenly into the numerator, the remainder (last row) must be zero and the Eqs. (25) and (26) must be satisfied:

$$a_1 - a_3\omega_n^2 = 2\zeta\omega_n(a_2 - 2a_3\zeta\omega_n) \tag{25}$$

$$K = a_0 + \omega_n^2(a_2 - 2a_3\zeta\omega_n) \tag{26}$$

So, Eq. (20) is decomposed into:

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)(a_3 s + \frac{a_0 + K}{\omega_n^2}) = 0 \tag{27}$$

Absolute stability is obtained for $\zeta = 0$ in Eqs. (25) and (27), and the result is the same as in Eq. (21). Using Eqs (25) and (26) for the design is easy. For the desired value of ζ the analytical solution of natural frequency is:

$$\omega_n = \frac{a_3\zeta + \sqrt{\zeta^2 a_1(a_2^2 - 4a_1)}}{4\zeta^2 - 1} \tag{28}$$

and the desired analytical solution of controlled gain as function of desired and process parameters is:

$$K = -a_0 + a_2\omega_n^2 - 2a_3\zeta\omega_n^3 \tag{29}$$

For the given damping factor ζ, using Eq. (28) one can calculate the appropriate undamped natural frequency ω_n. Substituting this frequency into Eq. (29) one obtains the desired result; the proportional controller gain that gives the desired damping factor in the closed--loop. The quotient provides the position of third pole.

The two Equations, (28) and (29) are the general algebraic and explicit solution to the problem of third order system and proportional controller. The problem is solved forever for all possible process parameters and closed loop specifications, ζ. We can also check how the result varies when we vary parameters.

Example 1

Consider the process of three isothermal reactors [14] with P controller. The parameters of the system are $a_0 = a_3 = 1$ and $a_1 = a_2 = 3$, and the Luyben's controller gain is $K = K_c/8$, where K_c is the gain of the Luyben's controller. The result of application the Eqs. (28) and (29) is shown in Table 1.

Table 1. Comparison of new analytical and Luyben's results for $\zeta = 0.316$ and P controller

Parameter	Analytical solution	Luyben solution
ω_n	1.157	1.176
K_c	16.31	17.00

The error is minimal and the results are practically identical.

Towill [15] gives the extensive explanation how the third-order model can be efficiently used for the approximation of higher order model including the models with dead time. For the higher order (4 and 5) model the functional dependences became highly complicated, and the use of these models is restricted to individual cases.

THE APPLICATION OF THE NEW METHOD FOR THE ANALYSIS OF THIRD ORDER PROCESS AND *PI* CONTROLLER

For the same open-loop transfer function as in Eq. (19) the closed-loop characteristic equation with PI controller is divided with the second order term (Eq. (30)):

$$\frac{a_3s^4 + a_2s^3 + a_1s^2 + (a_0 + K)s + \dfrac{K}{T_I}}{s^2 + 2\zeta\omega_n s + \omega_n^2} = a_3s^2 +$$
$$+ (a_2 - 2a_3\zeta\omega_n)s + a_1 - a_3\omega_n^2 - 2\zeta\omega_n(a_2 - 2a_3\zeta\omega_n) \quad (30)$$

To make remainder equal zero two equations must be satisfied:

$$K = \omega_n^2(a_2 - 2a_3\zeta\omega_n) +$$
$$+ 2\zeta\omega_n(a_1 - 2a_3\zeta\omega_n - 2\zeta\omega_n(a_2 - 2a_3\zeta\omega_n)) - a_0 \quad (31)$$

and

$$T_I = \frac{\omega_n^2(a_2 - 2\zeta\omega_n a_3)a_2\omega_n^2 + 2\zeta\omega_n(a_1 - a_3\omega_n^2 - 2\zeta\omega_n(a_2 - 2\zeta\omega_n a_3)) - a_0}{\omega_n^2(a_1 - a_3\omega_n^2 - 2\zeta\omega_n(a_2 - 2\zeta\omega_n a_3))} \quad (32)$$

The obtained equations are exactly what one need: the parameters solution of standard control problem as function of all given process parameters and desired closed-loop dominant poles. It is important to notice that this result for PI control is obtained easier than the result for P control. To get result for P control, it is necessary to solve the first equation, for the PI control this solutions is not necessary.

The performance criterion is simple: for the given damping factor of dominant poles find the biggest K and consequently biggest ω_n before system becomes unstable or not appropriate for some other reason. For the values of damping factor equal to zero one obtain two equations for ultimate values K_u and T_{Iu}.

For higher order PI processes, surprisingly, the solution can be also obtained analytically. The only problem is that the functions became more complicated. Again, there is no limit of system order (5) that can be efficiently used in the practice, except the number of terms in the two functions. This paper will now analyze another example.

Example 2

Consider the same example of three isothermal reactors as before Luyben [14], but now with PI controller. The parameters of this system models are $a_0 = a_3 = 1$ and $a_1 = a_2 = 3$, and the controller gain $K = K_c/8$.

Using Eqs. (31) and (32) for PI controller, we obtain the solutions for the desired damping coefficient. The results are compared with the Luyben's one in Table 2. The results for K_c and T_I are obtained for the value ω_n from Luyben's example.

Table 2. Comparison of new analytical and Luyben's result for $\zeta = 0.316$ for PI control design

Parameter	Analytical solution	Luyben solution
ω_n	1.06	1.064
K_c	15.54	15.00
T_I	4.52	4.50

The results are practically the same, the method is working properly and it is general, algebraic and explicit.

CONCLUSION

The proposed general method gives the analytical global solution for the design of the high order linear feedback control. This means, that not only the solution for one specified transfer function is obtained, but the solutions for all possible combinations of parameters of transfer function of the specified shape. For the P controller one gets the solution as two equations, which can be solved completely analytically for the open-loop transfer function of third, fourth and fifth order. For the *PI* controller the solution can be expressed analytically for any order, and the highest order is limited by the practical possibility of analytical derivation of long division.

The method gives a clear picture of absolute stability, response speed, and some measure of relative stability. The role of both parameters in PI controller, and the new limit of stability for T_I parameter is given. The simple method is derived for calculation the values of controller parameter in order to obtain the desired position of damping factor and the natural frequency. The simple explanation is given for the ultimate value of natural frequency and the method of calculation of this value and the desired value is derived. The results are expressed as real functions of real parameters.

The results presented in this paper are: general, which means that we have derived the general algebraic formulas for all possible first, second and third order system with P and PI controllers. To solve any first, second and third order problem you can use the derived algebraic formulas, insert your data for your process and controller specifications and design the controller using one explicit calculation.

Notations

P − Proportional
PI − Proportional-integral
a_i − Parameters of open-loop transfer functions ($i = 0,n$)
K − Controller gain
K_c − Controller gain in Luyben's example
$g(s)$ − Transfer function
p − Pole
s – Laplace transform variable
z − Zero
T_I − Controller integral time constant
ζ − Damping factor
ω_n − Undamped natural frequency
ω_{nu} − Ultimate natural frequency

Subscripts

u – Ultimate
n – Natural

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IZVOD

NOVI JEDNOSTAVAN ALGEBARSKI METOD GEOMETRIJSKOG MESTA KORENA ZA PROJEKTOVANJE SISTEMA AUTOMATSKOG UPRAVLJANJA

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Naučni rad

Prikazan je metod algebarske dekompozicije karakteristične jednačine da bi se uprostilo projektovanje zatvorenog kola za praktične primene. Metod se sastoji od dve dekompozicije. Prva dekompozicija je deljenje karakteristične jednačine na dve jednačine nižeg reda da bi se uprostilo projektovanje regulatora. Druga dekompozicija je zamena *s*-promenljive koja ima nejasan fizički smisao sa dve variable koje imaju potpuno jasan fizički smisao a to su faktor prigusenja, ζ, i prirodna frekvenca, ω_n. Ove dekompozicije uprošćavaju projektovanje sistema na pozicioniranje kompleksnih dominantnih polova u željene pozicije. U radu su izvedene sve jednačine za sisteme prvog, drugog i trećeg reda sa P i PI regulatorima, ukupno šest analitičkih jednačina. Slične jednačine mogu se izvesti analitički do jednačina petog reda, a pomoću kompjuterskih programa verovatno do *n*-tog reda. U slučaju sistema trećeg, četvrtog i petog reda karakteristične jednačine sa *P* i *PI* regulatorima mogu se dobiti kompletna analitička rešenja regulatora *K* kao funkcije željenog faktora prigušenja. Izvedeni metod ima tri važne prednosti u odnosu na standardne postupke. On je generalan, algebarski i eksplicitan, što nijedan standardni postupak nije. Ovi su rezultati matematički najbolji mogući.

Key words: Algebraic • Control Systems • Root Locus Analysis and Synthesis • Controller design

Ključne reči: Algebarski • Sistemi upravljanja • Položaj korena karakteristične jednačine • Projektovanje regulatora