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SCIENTIFIC PAPER

## APPLICATION OF NON-LINEAR DISCRETE-TIME FEEDBACK REGULATORS WITH ASSIGNABLE CLOSED – LOOP DYNAMICS

*In the present work the application of a new approach is demonstrated to a discrete-time state feedback regulator synthesis with feedback linearization and pole-placement for non-linear discrete-time systems. Under the simultaneous implementation of a non-linear coordinate transformation and a non-linear state feedback law computed through the solution of a system of non-linear functional equations, both the feedback linearization and pole-placement design objectives were accomplished. The non-linear state feedback regulator synthesis method was applied to a continuous stirred tank reactor (CSTR) under non-isothermal operating conditions that exhibits steady-state multiplicity. The control objective was to regulate the reactor at the middle unstable steady state by manipulating the rate of input heat in the reactor. Simulation studies were performed to evaluate the performance of the proposed non-linear state feedback regulator, as it was shown a non-linear state feedback regulator clearly outperformed a standard linear one, especially in the presence of adverse disturbance under which linear regulation at the unstable steady state was not feasible.*

The overriding idea in controller design approaches involves the use of feedback to explicitly shape the dynamic characteristics of the process by placing closed-loop poles at desirable locations on the complex plane (or assigning closed-loop dynamic modes/time constants) [1–5]. In other words the pole-placement objective is the central designing idea in control theory. For linear continuous and discrete dynamic process models, the synthesis of pole-placing state feedback control laws is very popular due to the straightforward algorithmic structure and intuitive appeal, since closed-loop poles are envisioned as tunable parameters [1, 3]. In particular, closed-loop poles are chosen to be faster than open-loop eigenvalues, in order to force the desired rate of decay of the system's state to steady-state values. However, by placing the eigenvalues of the closed-loop dynamics further in the left complex plane, high feedback gains might induce possible saturation problems or bring the system to instability due to model inaccuracies [1,3,6]. In practice, the tuning of closed-loop eigenvalues is performed by various ad-hoc techniques, trial and error approaches and heuristic rules [1,3,6].

Knowing that the majority of chemical and physical processes exhibit severe non-linear behavior, linear design techniques can not directly cope with process non-linearities since the customary approach is based on the linearization of a non-linear system around the equilibrium point and the subsequent use of linear design methodologies to attain the desired pole placement closed-loop specifications [2,5,7,8]. In order to overcome the limitations of linear design techniques and methods applied to non-linear processes,

non-linear control laws need to be synthesized that are capable of directly coping with the process non-linearities. Clearly the required objective would be the ability of the controller to control the process at a given designed steady state despite the presence of disturbances that may tend to drive the process state away from the design conditions. In particular, the control objective assumes stabilization of the process at the steady state of interest by synthesizing an appropriate regulator. There are two popular design approaches: 1) the exact input/output (I/O) feedback linearization approach, where non-linear state feedback regulation induces the linear I/O behavior of the process [2, 8–10]. This approach is restricted to minimum phase systems and regulation is understood in terms of forcing the process state to return to the design steady state and not in terms of regulating the specific output at a given set point value (or output tracking), 2) the geometric exact feedback linearization approach [11–14]. This approach is characterized by very restricted conditions that must be met by any physical system [2,5,8,15]. In the case of linear discrete-time systems, the feedback linearization problem with pole-placement has received attention because of the indisputable appeal of the direct digital implementation of advanced control algorithms in practice [16–19]. In all the mentioned design approaches, the feedback linearization with pole placement is achieved as the first step, by the simultaneous implementation of non-linear coordinate transformation that transforms the original non-linear system into a linear and controllable one, and in the second step, the usage of well-established linear pole-placement methodologies. However, the first step is based on a set of very restrictive conditions [16,18–20].

Motivated by the previous results, the present work demonstrates a new systematic approach to the discrete-time state feedback regulator synthesis

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problem developed by Kazantzis [21]. In particular, the objective of the present work was to demonstrate a design methodology that is not limited by restrictive conditions. The design methodology encompasses both the feedback linearization and the pole-placement objectives in a **single step** and the approach generalizes Luenberger's early ideas on a single-step design approach for pole-placement in linear systems [22]. The design method requires the formulation a problem in the context of the functional equations theory, where the design problem is translated into a problem of solving a system of non-linear functional equation (NFEs). The specific structure of the system of NFEs makes the problem solvable [23], as the solution of the system of NFEs is locally analytic, it allows the development of a series solution method that is easily programable with the aid of a symbolic software package such as MAPLE/MATLAB. Finally, the non-linear state feedback regulator synthesis method was applied to a continuous stirred tank reactor (CSTR) that exhibits steady-state multiplicity under non-isothermal operating conditions. The control objective was to regulate the reactor at the middle unstable steady state by manipulating the rate of input heat in the reactor. Simulation studies were performed to evaluate the performance of the proposed non-linear state feedback regulator.

## PRELIMINARIES

In this work we consider single-input non-linear discrete-time systems with the following state-space description:

$$x(k+1) = \Phi(x(k), u(k)) \quad (1)$$

where  $k = 0, 1, \dots$  is the discrete-time index,  $x(k) \in \mathbb{R}^n$  is the vector of state variables and  $u(k) \in \mathbb{R}$  the input variable. It is assumed, that  $\Phi(x, u)$  is a real analytic vector function on  $\mathbb{R}^n \times \mathbb{R}$ . Without loss of generality, suppose that the origin  $x = 0$  is an equilibrium point of (1), that corresponds to  $u = 0$ :  $\Phi(0, 0) = 0$ . Moreover, let  $F$  be the jacobian matrix of  $\Phi(x, u)$  evaluated at the equilibrium point  $x = 0$ :  $F = \frac{\partial \Phi}{\partial x}(0, 0)$  and  $G$  be the  $n \times 1$  vector:  $G = \frac{\partial \Phi}{\partial u}(0, 0)$ . Along this line, we assume that the  $n \times n$  controllability matrix,  $C = [G \mid FG \mid \dots \mid F^{n-1}G]$  has rank  $n$ .

Before proceeding to the exposition of the main result, the design steps in the standard formulation are briefly reviewed. In a first step a non-linear coordinate transformation  $z = T(x)$  can be found, and a static state feedback  $u = \Psi(x, u)$ , such that the original system Eq. 1 is transformed to a linear controllable one:

$$z(k+1) = Az(k) + bu(k) \quad (2)$$

where  $v$  is the external reference input. Since the transformed system is linear, the standard linear

pole-placement techniques may be employed. Clearly, the constant-gain vector  $K$  can be calculated, such that the state static feedback law:  $v = -Kz$ , when applied to Eq. 2, induces closed-loop dynamics:

$$z(k+1) = \bar{A}z(k) = (A - bK)z(k) \quad (3)$$

where  $\bar{A}$  is a matrix with prescribed eigenvalues. Another, alternative approach that was followed was initially developed by Luenberger, and it a single step approach, where a single-step simultaneous implementation of the linear coordinate transformation  $z = Tx$  is coupled with a linear state feedback:  $u = -Kz$  that induces the closed-loop dynamics:

$$z(k+1) = \bar{A}z(k) \quad (4)$$

where  $\bar{A}$  is a matrix with prescribed eigenvalues. Note that  $T$  is a constant transformation matrix and  $K$  is a constant gain vector. It can easily be seen that the transformation matrix  $T$  must satisfy the following matrix equation [16]:

$$TA - \bar{A}T = TbKT \quad (5)$$

or there exists an inverse transformation matrix  $W = T^{-1}$  that satisfies the following linear matrix equation:

$$AW - \bar{A}W = bK \quad (6)$$

If  $A$  and  $\bar{A}$  do not have common eigenvalues, the above equation admits a unique solution, which can be guaranteed by the full rank of the controllability pair  $(A, b)$  and the observability pair  $(K, A)$ . Therefore, once Eq. 5 or Eq. 6 is solved, the pole-placing state feedback is given by:  $u = -KTx$  and the aforementioned state feedback yield closed-loop dynamics as follows:

$$x(k+1) = T^{-1}\bar{A}Tx(k) \quad (7)$$

### 1.1. Problem formulation

Motivated by Luenberger's single-step approach for linear discrete-time systems, a non-linear analog was developed by Kazantzis and Dubljević [21], in order to bypass the restrictive conditions of the available exact feedback linearization approaches [16-18, 20]. In this approach, an associated system of non-linear functional equations (NFEs) of the following form was considered:

$$\begin{aligned} \omega(Az) &= \Phi(\omega(z), -cz) \\ \omega(0) &= 0 \end{aligned} \quad (8)$$

where  $\omega: \mathbb{R}^n \rightarrow \mathbb{R}^n$  is the unknown solution of Eq. 8, and  $A$  and  $c$  are constant matrices of appropriate dimensions. In order to avoid a cumbersome theoretical exposition that guarantees the solution to the above system of functional equations, we will refer to the work [21] where all necessary proofs and properties of systems (1) and (8) have been presented.

In order to demonstrate the used methodology in a concise way, we applied the linear differential operator

$\frac{\partial}{\partial z}$  to both sides of (8) and evaluated all the resulting quantities at the origin  $z = 0$ , and obtained:

$$\frac{\partial \omega}{\partial z}(0)A = F \frac{\partial \omega}{\partial z}(0) - Gc \quad (9)$$

The above is a Lyapunov matrix equation [4] with an unknown Jacobian:  $\frac{\partial \omega}{\partial z}(0)$  evaluated at  $z = 0$ .

Let us now consider the linear case given by the following expression:

$$\Phi(x(k), u(k)) = Fx(k) + Gu(k) \quad (10)$$

with  $F$ ,  $G$  being constant matrices with appropriate dimensions. We are looking for the control law  $u(k) = -cz(k)$  that will induce the following closed-loop dynamics  $z(k+1) = \bar{A}z(k+1)$ . The proposed linear transformation  $z = Sx$  gives:

$$\begin{aligned} Sx(k+1) &= SFx(k) - SGcz(k) \\ x(k+1) &= \bar{A}z(k) \end{aligned} \quad (11)$$

which yields

$$SF - \bar{A}S = ScGS \quad (12)$$

or after the transformation  $T = S^{-1}$ , one obtains the following Lyapunov matrix equation:

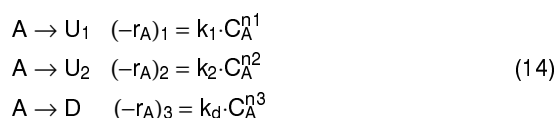
$$FT - \bar{A}T = cG \quad (13)$$

The above Lyapunov equation admits a unique and invertible solution  $T$ .

In the non-linear case the proposed transformation is non-linear mapping  $z = S^{-1}(x) = \bar{S}(x)$ , which is obtained by the series solution method. Namely, by the expansion of the vector functions  $\Phi(x, u)$  and  $S(x)$  in a Taylor series and by substituting into the appropriate NFEs by matching the coefficients of the same order the desired solution can be obtained [21].

### Main result

In order to illustrate the main aspects of the proposed non-linear discrete-time state feedback regulator synthesis method, a representative chemical reactor example is considered in this section. We considered an ideal continuous stirred tank reactor (CSTR) in non-isothermal operation, where the following exothermic irreversible reaction takes place [10]:



The capital letters  $A$ ,  $U_1$ ,  $U_2$  and  $D$  denote the following:  $A$  is the reactant,  $U_1$  and  $U_2$  are undesirable side products and  $D$  is the desired product. The dependence of the reaction rate constants  $k_1$ ,  $k_2$  and  $k_d$  on temperature is given by the Arrhenius equation  $k_i =$

Table 1. Kinetic and thermodynamic parameters which define reactions (Eq. 14)

Process parameters	Values
$Z_1$	$2 \cdot 10^3 \text{ m}^6 \text{ kmol}^{-2} \text{ s}^{-1}$
$Z_2$	$3.4 \cdot 10^6 \text{ kmol}^{0.5} \text{ m}^{-1.5} \text{ s}^{-1}$
$Z_3$	$2.63 \cdot 10^5 \text{ s}^{-1}$
$(-\Delta H_1)$	$4.5 \cdot 10^4 \text{ kJ kmol}^{-1}$
$(-\Delta H_2)$	$5 \cdot 10^4 \text{ kJ kmol}^{-1}$
$(-\Delta H_3)$	$6 \cdot 10^4 \text{ kJ kmol}^{-1}$
$E_{a1}$	$4.9 \cdot 10^4 \text{ kJ kmol}^{-1}$
$E_{a2}$	$6.5 \cdot 10^4 \text{ kJ kmol}^{-1}$
$E_{ad}$	$5.7 \cdot 10^4 \text{ kJ kmol}^{-1}$
$n_1$	3
$n_2$	0.5
$n_3$	1
$R$	$8.354 \text{ kJ kmol}^{-1} \text{ K}^{-1}$
$\rho$	$10^3 \text{ kg m}^{-3}$
$c_p$	$4.2 \text{ kJ kg}^{-1} \text{ K}^{-1}$

$Z_i \exp(-E_{ai}/RT)$ ,  $i = 1, 2$  and  $k_d = Z_d \exp(-E_{ad}/RT)$ , and the parameters are given in Table 1.

The energy and mass balances render the following non-linear dynamical model of the reactor:

$$\begin{aligned} \frac{dC_A}{dt} &= -k_1 C_A^3 - k_2 C_A^{0.5} - k_d C_A \\ \frac{dT}{dt} &= \frac{1}{\rho c_p} \left( (-\Delta H_1) k_1 C_A^3 + (-\Delta H_2) k_2 C_A^{0.5} + (-\Delta H_d) k_d C_A \right) \\ &+ \frac{1}{\tau} (T_i - T) + \frac{q}{\rho c_p V} \end{aligned} \quad (15)$$

where  $q$  is the rate of heat input to the reactor. It is assumed that the reactor temperature,  $T$ , is a measurable output and  $C_A$  and  $q$  are measurable inputs. Experimental investigations have demonstrated the multiplicity of steady states for the reaction system. For the value of  $q = 1.030 \text{ KJs}^{-1}$  there are three steady states and elementary stability analysis using Lyapunov's first method, implies that the upper and lower steady states are stable, whereas the middle one is unstable. The control objective was to regulate the system around the unstable steady state where  $(C_A, ss; T_{ss}) = (3.3 \text{ kmol m}^{-3}, 370 \text{ K})$ , by manipulating the heat input into the reactor  $\frac{q}{\rho c_p V}$ . It is at middle unstable steady state where both conversion and temperature acquire reasonable values, and therefore, this steady state is operationally favorable. For the value of  $q = -1.030 \text{ KJ s}^{-1}$  there are three steady states:

$$\begin{aligned} SS_1 : (C_A, ss, T_{ss}) &= (8.0 \text{ kmol m}^{-3}; 310 \text{ K}) \\ SS_2 : (C_A, ss, T_{ss}) &= (3.3 \text{ kmol m}^{-3}; 370 \text{ K}) \\ SS_3 : (C_A, ss, T_{ss}) &= (1.3 \text{ kmol m}^{-3}; 400 \text{ K}) \end{aligned} \quad (16)$$

and, shown in Figure 1, the dynamic simulation of the autonomous system dynamics for different initial values,  $SS_1$  and  $SS_3$  are stable, while the  $SS_2$  is unstable. As a consequence of different steady states, linearized models and associated eigenvalue distributions will admit different values. Namely, as expected, one of the eigenvalues of the linearized system with respect to the middle steady states is positive:

$$\begin{aligned} SS_1: (\lambda_1, \lambda_2 &= -3.439 \cdot 10^{-3}; -9.143 \cdot 10^{-4}) \\ SS_2: (\lambda_1, \lambda_2 &= -5.825 \cdot 10^{-3}; 7.721 \cdot 10^{-4}) \\ SS_3: (\lambda_1, \lambda_2 &= -6.376 \cdot 10^{-3}; -3.288 \cdot 10^{-4}) \end{aligned} \quad (17)$$

Accurate non-linear sample data representation of Eq. (15) can be easily obtained on the basis of Euler's discretization method with the sampling period given as  $\delta = \frac{1}{\|\lambda_{\max}\|}$ . So the discrete time non-linear system takes the following form:

$$\begin{aligned} C_A(n+1) &= \Phi_1(C_A(n), T(n), U(n)) \\ T(n+1) &= \Phi_2(C_A(n), T(n), U(n)) \end{aligned} \quad (18)$$

where

$$\begin{aligned} \Phi_1 = C_A(n) + \delta \left\{ -Z_1 e^{\frac{E_{a1}}{RT(n)}} C_A(n)^{n_1} - \right. \\ \left. - Z_2 e^{\frac{E_{a2}}{RT(n)}} C_A(n)^{n_2} - Z_d e^{\frac{E_{ad}}{RT(n)}} C_A(n)^{n_d} + \right. \\ \left. + \frac{C_{Ai} - C_A(n)}{\tau} \right\} \end{aligned}$$

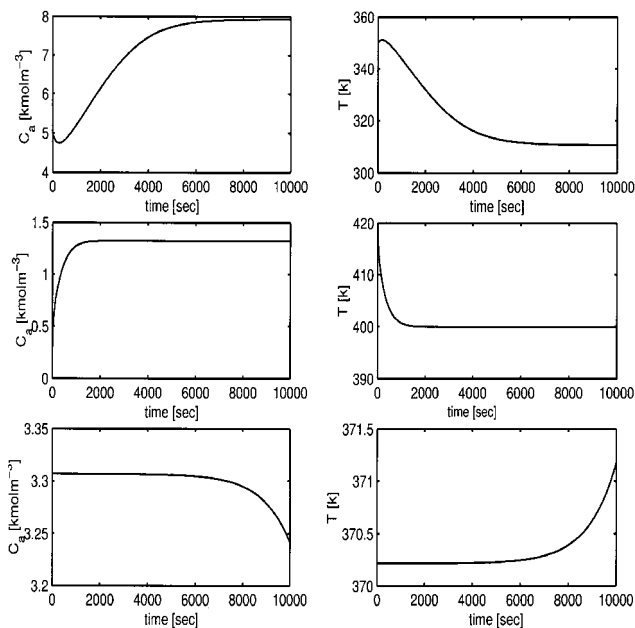


Figure 1. Simulation of autonomous dynamics, Eq. 15 for various initial conditions

$$\begin{aligned} \Phi_2 = T(n) + \delta \left\{ ((-\Delta H_1) Z_1 e^{\frac{-E_{a1}}{RT(n)}} C_A(n)^{n_1} + \right. \\ \left. + (-\Delta H_2) Z_2 e^{\frac{-E_{a2}}{RT(n)}} C_A(n)^{n_2} + (-\Delta H_d) Z_d e^{\frac{-E_{ad}}{RT(n)}} C_A(n)^{n_d} \right) \frac{1}{\rho C_p} + \\ \left. + \frac{T_i - T(n)}{\tau} + \frac{q}{\rho C_p V}(n) \right\} \end{aligned} \quad (19)$$

and note that  $U(n) = \frac{q}{\rho C_p V}(n)$ . Notice that, under the discretization method employed, the equilibrium and stability characteristics of the system have not changed. For simplicity reasons, let us now define the deviation variables with respect to the steady state of interest:

$$\begin{aligned} x_1 &= C_A - C_{A,ss} \\ x_2 &= T - T_{ss} \\ u &= U - U_{ss} \end{aligned} \quad (20)$$

and denote:  $\bar{\Phi}_{(i)}(x_1, x_2, u) = \Phi_{(i)}(C_{A,ss} + x_1, T_{ss} + x_2, U_{ss} + u)$ . According to the proposed synthesis method, one must select an observable pair of matrices  $c, A$ . For the specific example we choose matrix  $A$  to be stable with desirable dynamic characteristics:

$$c = [c_1 \quad c_2] = [1 \quad 0] \quad (21)$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0.92 & 25 \\ 0 & 0.98 \end{bmatrix} \quad (22)$$

Under the above selection, the system of NFE's admits a unique analytic and invertible solution in the neighborhood of the middle unstable steady state of interest  $(x_{1,s}, x_{2,s}) = (0, 0)$ . In particular, a series solution of the following system of NFE's is sought:

$$\begin{aligned} \bar{\omega}_1(\Phi_1(x_1, x_2, -\sum_{j=1}^2 c_j \bar{\omega}_j), \Phi_2(x_1, x_2, -\sum_{j=1}^2 c_j \bar{\omega}_j)) = \sum_{i=1}^2 (a_{1i} \bar{\omega}_i) \\ \bar{\omega}_2(\Phi_1(x_1, x_2, -\sum_{j=1}^2 c_j \bar{\omega}_j), \Phi_2(x_1, x_2, -\sum_{j=1}^2 c_j \bar{\omega}_j)) = \sum_{i=1}^2 (a_{2i} \bar{\omega}_i) \end{aligned} \quad (23)$$

Higher-order Taylor coefficients of the unknown solution can be easily computed using a simple MAPLE code, whereas the linear terms can be obtained as a solution of the Lyapunov matrix equation. Particularly, a second-order truncation of the Taylor series expansion of the solution of the system of NFE's, Eq. 23, gives rise to the following non-linear discrete-time state feedback regulator:

$$u = cW = -\sum_{j=1}^2 c_j \bar{\omega}_j = -Z_1 \quad (24)$$

where

$$\begin{aligned} Z_1 &= d_1 x_1 + d_2 x_2 + d_3 x_1^2 + d_4 x_1 x_2 + d_5 x_2^2 \\ Z_2 &= g_1 x_1 + g_2 x_2 + g_3 x_1^2 + g_4 x_1 x_2 + g_5 x_2^2 \end{aligned} \quad (25)$$

with calculated coefficients up to second-order:

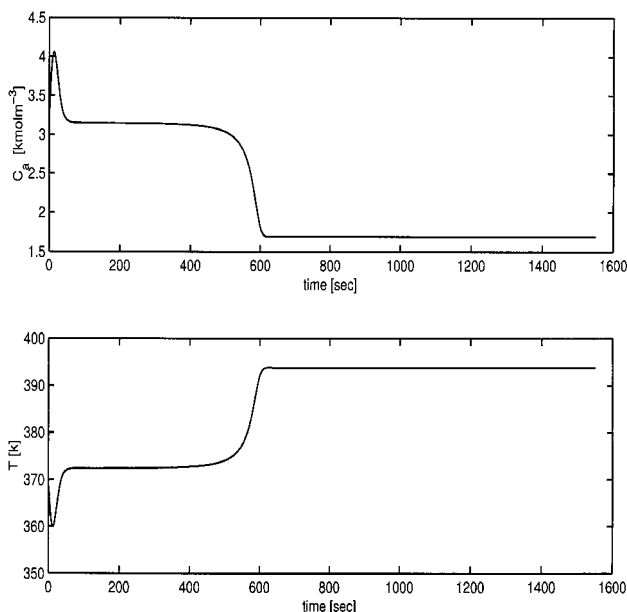


Figure 2. Closed-loop response under a linear state feedback regulator

$$d_1 = -2.403403392$$

$$d_2 = -0.1578638901$$

$$d_3 = -2.09321702$$

$$d_4 = -0.3258964545$$

$$d_5 = -0.01272694127$$

$$g_1 = 0.01036984786$$

$$g_2 = 0.7962728292 \cdot 10^{-3}$$

$$g_3 = 0.9024245418 \cdot 10^{-2}$$

$$g_4 = 0.1405020588 \cdot 10^{-2}$$

$$g_5 = 0.5486955467 \cdot 10^{-4}$$

For the particular initial point selected ( $x_1, x_2$ ) = (0.02 - 1.68), Figures 2 and 3 show the closed-loop response with a linear and quadratic regulator, respectively. The initial condition selected represents large deviations from the unstable middle steady state and they can be viewed as large disturbances that upset the process. It is shown that for this "adverse" initial condition, the linear state feedback regulator does not exhibit adequate strength to derive the system back to the middle unstable steady state because it does not capture the dominant non-linearities of the dynamic process model. Figure 2 indicates that the second order discrete-time state feedback control law [24] successfully performs regulation of the system at the middle unstable steady state in the presence of adverse "initial" initial conditions. It may be concluded that the second-order state feedback regulator is capable of capturing the dominant process non-linearities and bringing the system back to the desirable middle unstable steady state. The representative simulations demonstrate that a non-linear time-discrete state feedback regulator outperforms a standard linear one in

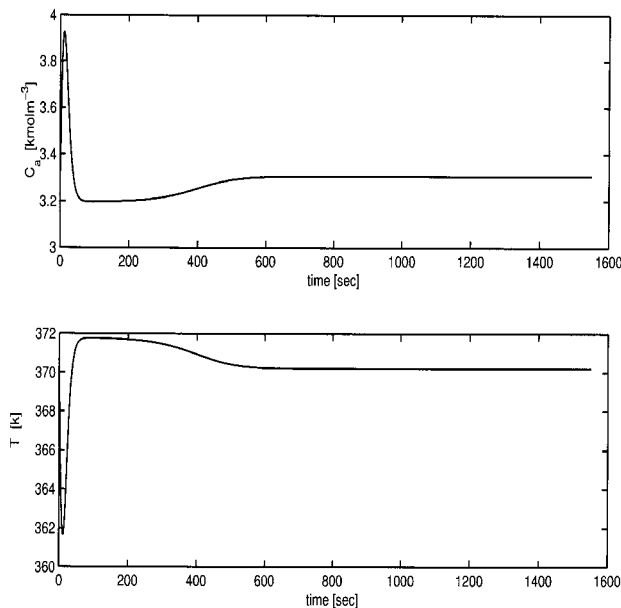


Figure 3. Closed-loop response under a quadratic state feedback regulator

the presence of a set of adverse initial conditions, under which linear regulation at the middle unstable steady state is not always feasible.

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## IZVOD

### APLIKACIJA NELINEARNOG DISKRETNOG REGULATORA POVROTNE SPREGE SA PRESPECIFICIRANOM DINAMIKOM

(Naučni rad)

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Osnovna ideja upravljanja u procesnoj industriji se bazira na korišćenju povratne sprege da bi se modifikovala dinamika procesa i/ili zadovoljile zahtevane proizvodne specifikacije. Popularan i jednostavan dizajn regulatora za hemijske procese, koji su predstavljeni linearnim kontinualnim vremenski invarijantnim sistemima i linearnim diskretnim vremenskim invarijantnim sistemima podrazumeva eksplicitno postavljanje polova sistema, zatvorenog povratnog sprega na željena mesta u kompleksnoj ravni. Ipak, kako je većina hemijskih procesnih sistema inherentno nelinearna to može proizvesti nedopustivo loše radne performanse regulatora koje su posledica linearnih dizajn metodologija. Kako bi prevazišli nedostatke ovih metodologija neophodno je predvideti i realizovati nelinearne regulatore povratne sprege koji bi bili u mogućnosti da se nose sa inherentnim nelinearnostima u procesu i da održavaju procesne parametre na prespecificiranom željenom stacionarnom stanju uprkos različitim poremećajima kojima proces može da bude izložen. U ovom radu razmatra se primena nove nelinearne dizajn metodologije na diskretnom nelinearnom sistemu pri čemu je istovremeno izvršena linearizacija sa povratnom spregom i postavljanje polova sistema. Istovremeno implementacija nelinearne kordinatne transformacije i nelinearnog zakona povratne sprege je realizovana rešavanjem sistema nelinearnih jednačina sistema na primeru protočnog reaktora sa idealnim mešanjem (PRIM) koji radi u neizotermkim uslovima kada postoje višestruka stacionarna stanja. Cilj regulacije je održavanje rada reaktora u nestabilnom stacionarnom stanju koristeći brzinu dovođenja toplote u reaktor kao manipulativnu promenljivu. Intenzivna simulacija pokazuje da je nelinearni regulator u potpunosti superiorniji od linearnog i to posebno u slučaju snažnih poremećaja kada linearni regulator nije u stanju da izvrši stabilizaciju procesa.

Ključne reči: Nelinearni sistemi • Diskretni sistemi • Linearizacija povratnom spregom • Podešavanje polova •  
Key words: Non-linear Systems • Discrete-time systems • Feedback linearization • Pole-placement •