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MICROBIAL COMPETITION IN BIOREACTORS

Competition for nutrients and other resources is an interaction common among microbial species growing together in the same environment. Such an environment can be created in the laboratory in a bioreactor in order to study this type of interaction and its effect on the microorganisms. Competition tends to eliminate species from the system. The main question then is whether the competing microbial species can coexist and under what conditions. The number of nutrients for which the microorganisms compete plays an important role, while periodic oscillations and spatial heterogeneity have a favorable effect on coexistence.

Key words: Bioreactor, Microbial fermentation, Mixed cultures, Competition.

Mixed cultures of microorganisms are often studied in bioreactors. Bioreactors are considered as crude models of natural ecosystems and they are used to study, under controlled laboratory conditions, the various types of interactions that arise among microbial species inhabiting the same environment. These interactions may be direct, when there is contact of the different species with each other, or indirect, when the interactions are exerted by altering the abiotic environment. Competition for nutrients and other resources is the most common and the most important of the indirect interactions. It is almost always present in systems of mixed microbial populations. Fredrickson and Stephanopoulos [1] defined competition and classified its various types. The interaction of competition between two microbial populations arises when there is at least one resource consumed by both populations which affects the growth rate of at least one of them. The definition can be also expressed by using a set theoretic approach. Specifically, we define as S_1 the set of all the resources consumed by population 1, S_2 the set of all the resources consumed by population 2, S_1' the set of all the resources consumed by population 1 and affecting its growth rate, S_2' the set of all the resources consumed by population 2 and affecting its growth rate. Apparently $S_1' \subseteq S_1$ and $S_2' \subseteq S_2$. We have competition when $S_1 \cap S_2 \neq \emptyset$ or $S_1 \cap S_2' \neq \emptyset$.

Depending on the resources belonging in each of the above sets, there are various types of competition. One way of classification is based on the number of resources competed for. The competition is called single, double, etc. if the populations compete for one, two, etc. resources. We say that we have total competition when the populations compete for all the

resources that affect the growth rate of at least one of the populations. Otherwise, the competition is called partial. A special case, which is of considerable interest, is total competition which is also single, *i.e.*, there is only one resource affecting the growth rate of the two populations and is consumed by them. This type of competition is called simple. Finally, when there are no other interactions present, but competition is the only interaction, the competition is called pure. The simplest type of competitive interaction is pure and simple competition.

There is a considerable body of theoretical and experimental work on pure and simple competition, as well as, competition for more than one resource. An overview of this work is presented and discussed here. First, the case of pure and simple competition in a homogeneous and time-invariant environment is presented, and then the effects of time-variation of the conditions and of spatial heterogeneity of the environment are examined. Then, competition for two or more resources is presented and discussed. All cases presented here concern competition for chemicals and not competition for biotic resources, the dynamics of which is complicated by growth of the resources consumed. Also, cases where competition is not pure, but other types of interactions are also involved, are not considered, although there are several studies concerning such systems.

PURE AND SIMPLE COMPETITION

Pure and simple competition will be first examined in a homogeneous and time-invariant environment. Such an environment can be created in controlled laboratory conditions in an ideal chemostat with constant operating conditions. A chemostat is a well-mixed vessel of continuous operation, in which the microbial species grow and which is fed with medium for growth of the species in steady flow. Control devices for temperature, pH, etc. ensure operation under constant time-invariant conditions.

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Competition in a chemostat

A model of pure and simple competition in a chemostat was first introduced and analyzed by Powell [2]. The equations of the model are:

$$\frac{dx_1}{dt} = -Dx_1 + \mu_1(s)x_1 \quad (1a)$$

$$\frac{dx_2}{dt} = -Dx_2 + \mu_2(s)x_2 \quad (1b)$$

$$\frac{ds}{dt} = D(s_F - s) - \frac{1}{Y_1} \mu_1(s)x_1 - \frac{1}{Y_2} \mu_2(s)x_2 \quad (1c)$$

where x_1 and x_2 are the biomass concentrations of microbial populations 1 and 2, respectively, s is the concentration of the rate-limiting nutrient, D is the chemostat dilution rate, s_F is the concentration of the rate-limiting nutrient in the feed, Y_1 and Y_2 are the yield coefficients of the two populations, and μ_1 and μ_2 are the specific growth rates of the two populations. Monod's model [3] is used for the specific growth rates of microbial populations:

$$\mu_i(s) = \frac{\mu_{mi}s}{K_i + s} \quad i=1,2 \quad (2)$$

The system of equations (1) has four possible steady states:

1. Total washout:
 $x_1 = 0, x_2 = 0, s = s_F$.
2. Growth of only population 1:
 $x_1 > 0, x_2 = 0, s_F > s > 0$.
3. Growth of only population 2:
 $x_1 = 0, x_2 > 0, s_F > s > 0$.
4. Coexistence of the two populations:
 $x_1 > 0, x_2 > 0, s_F > s > 0$.

From equations (1) it is deduced that a necessary condition for the coexistence steady state to be possible is the existence of a positive value s_c such that

$$\mu_1(s_c) = \mu_2(s_c) \quad (3)$$

Namely, the curves described by equations (2) must cross at a positive s value, as shown in Figure 1b. On the contrary, for the case shown in Figure 1a, coexistence is not possible under any conditions. The

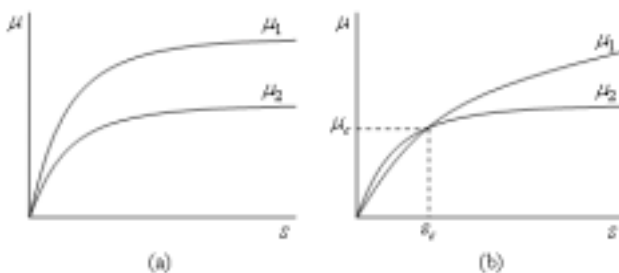


Figure 1. Relative position of the specific growth rate curves of the two microorganisms

value of the nutrient concentration at the point of intersection of the curves is:

$$s_c = \frac{\mu_{m2}K_1 - \mu_{m1}K_2}{\mu_{m1} - \mu_{m2}} \quad (4)$$

Therefore, the values of the parameters of the specific growth rates [eq. (2)] must be such that the expression in eq. (4) is positive. The common value of the specific growth rates at s_c is:

$$\mu_c = \frac{\mu_{m2}K_1 - \mu_{m1}K_2}{K_1 - K_2} \quad (5)$$

One more condition that must be satisfied for the coexistence steady state to be possible in the chemostat is the dilution rate D to be equal to the critical value (c) of the specific growth rate. Namely, there is just a single value of the dilution rate where coexistence is possible.

Analyzing the dynamics of the system of equations (1), we can construct the operating diagram of the system, which shows the effect of the operating parameters of system D and s_F on the long-term behavior of the chemostat [4]. The operating diagram for both cases of Figure 1 is shown in Figure 2. In region I of the diagram total washout is observed, in regions II and III population 1 dominates, and in regions IV and V population 2 dominates. As can be seen from Figure 2a, for the case where the specific growth rate curves do not cross, there are no conditions for growth of the population with the lower specific growth rate, in this case population 2. In the operating diagram shown in Figure 2b coexistence is obtained only for conditions lying on the horizontal line separating regions III and IV. Namely, it must be that $D = \mu_c$ and $s_F > s_c$. At the coexistence steady state the population concentrations x_1 and x_2 are not uniquely determined, but all pairs of values satisfying the following equation

$$s_F = s_c + \frac{1}{Y_1} x_1 + \frac{1}{Y_2} x_2 \quad (6)$$

are steady states. In the phase space the coexistence steady state is not a point but a straight line, as depicted in Figure 3. At the coexistence steady state, the system has two negative eigenvalues and one eigenvalue equal to 0. The zero eigenvalue is characteristic of the free mo-

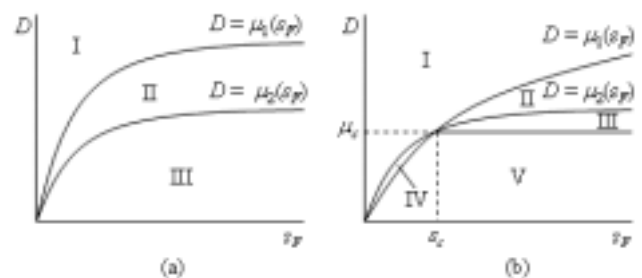


Figure 2. Operating diagram of the system of eq. (1)

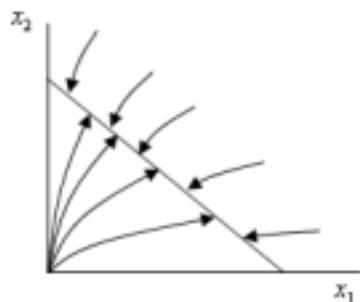


Figure 3. Line of semi-stable coexistence steady states

tion of the state of the system along the line. Since the other two eigenvalues are negative, the system always ends up at one of the infinite coexistence steady states on the line. Each one of these steady states is characterized as semi-stable [5]. The system there is structurally unstable [6] and even the slightest change in a parameter value qualitatively alters its dynamic behavior.

If instead of Monod's model we use Andrews's nutrient-inhibition model [7] for the specific growth rates of the two populations:

$$\mu_i(s) = \frac{\mu_{mi}s}{K_i + s + \frac{s^2}{K_i}} \quad i = 1, 2 \quad (7)$$

then things are more complicated, as shown by Aris and Humphrey [5]. Depending on the location of the specific growth rate curves relative to each other, there are several new possibilities. There is the possibility of the existence of up to two steady states where one of the two populations dominates, but only one is stable. Moreover, it is possible for some values of D that there exists one stable steady state where population 1 dominates and one stable steady state where population 2 dominates, and thus which population prevails at the end depends not only on the operating conditions, but also on the initial conditions of the system. Also, since the specific growth rate curves can have up to two intersection points corresponding to positive s values, there can be two critical values of D where coexistence is obtained. If at the intersection point the slopes of the curves are both positive, as in Figure 1b, the coexistence steady state is semi-stable. If the slopes of the two curves are both negative, the coexistence steady state lying on the line of steady states have one positive, one negative and one zero eigenvalue, and they are characterized as semi-unstable, since there is a direction along which the system moves away from the steady state (Figure 4a). In the case where the two curves cross with opposite slopes, a part of the line of steady states is semi-stable and a part is semi-unstable (Figure 4b). At the intermediate point between the two parts of the line the system has a double zero eigenvalue. The operating diagrams for all the cases of the relative location of the two spe-

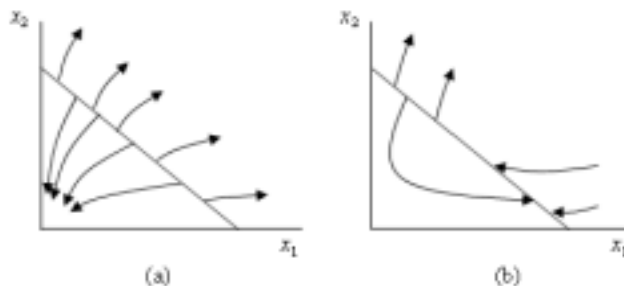


Figure 4. Line of coexistence steady states when the specific growth rate curves cross with (a) decreasing slopes (semi-unstable steady states), (b) opposite slopes (partly semi-stable and partly semi-unstable line of steady states)

cific growth rate curves have been given by Lenas and Pavlou [8].

A general conclusion is that in pure and simple competition in a chemostat, irrespective of the particular expressions for the specific growth rates [9–11], coexistence in a steady state is achieved only for specific discrete values of the dilution rate. Thus the chemostat must operate at exactly one of these values for the two populations to grow together. Otherwise, not more than one can grow in the chemostat. In general, for any number of microbial populations involved in pure competition for a single nutrient in the chemostat, not more than one can establish itself. The one that prevails at the end is the one having the competitive advantage, i.e., the lowest break-even nutrient concentration [12,13]. The break-even concentration for the i -th population is the smallest positive root of the equation:

$$D = \mu_i(s) \quad (8)$$

The coexistence of two competing populations in a chemostat is only a theoretical possibility. In practice, operating the chemostat at exactly the required critical value of the dilution rate is not possible, since there will always be random fluctuations entering the system. Stephanopoulos et al. [14] did a stochastic analysis of the chemostat with two competing microbial populations following Monod's model for their growth, in order to study the effect of random fluctuations in its operation. Specifically, they assumed that the dilution rate was subjected to fluctuations that could be described by white noise:

$$D(t) = D_0 + w(t) \quad (9)$$

where D_0 is the mean value of the dilution rate. The basic conclusions of their analysis are:

1. Given enough time of chemostat operation, it is certain that one of the two populations will be extinct, even if the mean value of the dilution rate is equal to the critical value, $D_0 = \mu_c$.

2. There is a finite probability for extinction of any of the two populations even when $D_0 \neq \mu_c$, i.e., even

when one of the two populations has the competitive advantage.

3. If the mean value of the dilution rate D_0 is close to the critical value μ_c and the intensity of the noise is low, a very long time is required for extinction of one of the populations.

The first of the above conclusions practically means that fluctuations will eventually cause washout of one the two microorganisms even if we try to operate the chemostat at exactly the critical value of the dilution rate. The second of the conclusions states that, even if we set the dilution rate such that one population has the competitive advantage, fluctuations could lead to its extinction. The third of the conclusions is of practical significance, since it shows that we can operate the chemostat keeping the two populations for a sufficiently long time.

Pure and simple competition in a chemostat has been also the subject of several experimental studies [15–24], which verify qualitatively the predictions of the theoretical analysis. Namely, they show that with appropriate choice of the operating conditions, we can select which microbial species will prevail in the system, and the coexistence of two species is not possible.

The basic practical conclusion of the study of pure and simple competition of two microbial populations in a chemostat is that the two populations cannot coexist, but one of the two will be extinct from the system. This is a variation of the competitive exclusion principle proposed by Hardin [25]. From the competitive exclusion principle the question arises how it is possible for so many species of phytoplankton competing for a limited number of nutrients to coexist in a relative homogeneous environment. This is the so-called paradox of the plankton put forward by Hutchinson [26]. Hutchinson offered the explanation that coexistence of the competing populations is made possible by the periodic change of the conditions of the system due to seasonal variations. Periodic variation of the conditions gives the competitive advantage to each of the population for a period of time and allows them to coexist. In a chemostat this can be accomplished by periodic variation of one of the operating parameters.

Effect of periodically varying conditions

For a system of two microbial populations involved in pure and simple competition in a chemostat, one can vary periodically the operating parameters D and s_F . The variation can be sinusoidal, orthogonal or any other type. Stephanopoulos et al. [27] demonstrated numerically that periodic variation of either of the operating parameters in the system of equations (1) can lead to stable coexistence of the two microbial populations. The specific growth rate curves must cross (case in Figure 1b), so that with periodic variation the competitive advantage alternates between the two populations. Coexistence, of course, is obtained not in a

steady state, but in a state of sustained oscillations (stable limit cycle). The same researchers also examined the case of periodic harvesting of part of the culture in the reactor and its replacement by fresh medium. They also found stable coexistence of the two populations. Coexistence through harvesting was also demonstrated by Stewart and Levin [28], who considered the case of removing a constant fraction of the culture and replacing it by fresh medium, when all the rate-limiting nutrient was consumed. Matsubara et al. [29] did a bifurcation analysis of the same system for periodic variation of D and determined conditions for coexistence. Butler et al. [30] analyzed mathematically the case of D variation, whereas Hsu [31], Smith [32] and Hale and Somolinos [33] analyzed mathematically the case of s_F variation. They proved analytic results regarding the conditions for stable coexistence. Lenas and Pavlou [8] did a numerical bifurcation analysis of the case of periodic variation of D when growth of the populations follows, not only Monod's model, but also Andrews's model. The type of periodic variation they used was sinusoidal:

$$D(t) = D_0 + a \cos \omega t \quad (10)$$

They constructed operating diagrams for the different cases of crossing of the specific growth rate curves. Since now there exist four operating parameters D_0 , a , ω , s_F , they kept constant the amplitude a and the frequency ω of variation and constructed diagrams with coordinates D_0 and s_F . The operating diagram for the case where both specific growth rates are described by Monod's model is shown in Figure 5. In regions I–V the outcome of the system is analogous to the one in the corresponding regions in Figure 2b, but now the behavior is periodic and not steady state. Namely, in region I total washout is observed, in regions II and III population 1 dominates, and in regions IV and V population 2 dominates. In region VI we have stable coexistence in the form of sustained oscillations (limit cycle). We see that coexistence is obtained for a range of values of the operating parameters and not for discrete values of them. The system is structurally stable. In the case where the specific growth curves are of the Andrews-type and they cross with opposite slopes, coexistence can be observed, not only in a

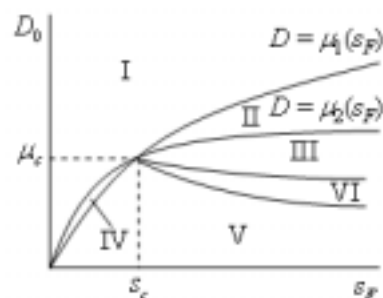


Figure 5. Operating diagram of the chemostat with periodically varying dilution rate $D(t) = D_0 + a \cos \omega t$.

periodic state, but also in a quasi-periodic or in a chaotic state. This is due to perturbation of the point with double zero eigenvalue on the line of steady states of the unperturbed system (Figure 4b).

Although there are many theoretical studies showing the stable coexistence of pure and simple competitors in periodically operated bioreactors, there are only two studies that demonstrate this experimentally. Davison and Stephanopoulos [34] studied the system *Saccharomyces cerevisiae* and *Escherichia coli* in a chemostat with glucose as their common rate-limiting nutrient. They showed that with periodic variation of the pH they were able to achieve stable periodic coexistence in the reactor. Dikshitulu et al. [35] studied a system of *Pseudomonas putida* and *Pseudomonas resinovorans* with phenol as their common rate-limiting nutrient in a sequencing fed-batch reactor (SFBR), which may be considered as equivalent to a chemostat with periodically varied dilution rate. The particular system is of interest in waste treatment applications. A mathematical model of the system was developed and analyzed and its predictions were compared with experimental results. Very good agreement was found and stable coexistence of the two species was demonstrated both numerically and experimentally.

All of the above studies assume that the specific growth rate curves cross at a positive value of the limiting nutrient concentration, so that for each population there are concentration values where it grows faster and thus has the competitive advantage. The next question is whether for the case where the specific growth rate curves do not cross (case in Figure 1a) there is a way by periodic operation of the chemostat to give the competitive advantage to the more slowly growing population. Stephens and Lyberatos [36] and Pavlou et al. [37] examined whether the delay in the response of the microorganisms to environmental variations and, specifically, to changes in the nutrient concentration can give such an advantage to the slower growing species. They used the following model equations:

$$\frac{dx_1}{dt} = -Dx_1 + \frac{\mu_{m1}z_1}{K_1 + z_1} x_1 \quad (11a)$$

$$\frac{dx_2}{dt} = -Dx_2 + \frac{\mu_{m2}z_2}{K_2 + z_2} x_2 \quad (11b)$$

$$\frac{ds}{dt} = D(s_F - s) - \frac{1}{Y_1} \frac{\mu_{m1}s}{K_1 + s} x_1 - \frac{1}{Y_2} \frac{\mu_{m2}s}{K_2 + s} x_2 \quad (11c)$$

$$\frac{dz_1}{dt} = \alpha_1 (s - z_1) \quad (11d)$$

$$\frac{dz_2}{dt} = \alpha_2 (s - z_2) \quad (11e)$$

The delay in the response of the organisms to changes in the nutrient concentration is introduced with the auxiliary variables z_1 and z_2 , on which the specific

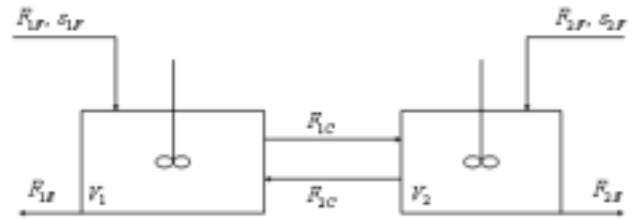


Figure 6. Configuration of two interconnected chemostats

growth rates are assumed to depend and not directly on the nutrient concentration s . The variables z_1 and z_2 do not have physical meaning, but they help describe the phenomenon of the delay. At steady state z_1 and z_2 are equal to s , but at transient conditions, when the concentrations change with time, there is a difference, which expresses the delay in the response. This difference depends on the values of the adaptability coefficients α_1 and α_2 , which represent how fast the microorganisms adapt to changes in the nutrient concentration. The smaller these coefficients are, the bigger the delay. Analysis of the model equations (11) showed that coexistence of the two populations is obtained in a periodic state with periodic variation of the nutrient concentration in the feed even when the specific growth rate curves do not cross, as long as the population with the highest specific growth rate also has the highest adaptability. It appears that low adaptability of a population gives it a competitive advantage.

Having established the fact that periodic operation of the chemostat can lead to the stable coexistence of two microbial populations involved in pure and simple competition, one would ask if this is possible with more than two populations. Lenas and Pavlou [38] studied the case of pure and simple competition of three microbial populations in a chemostat with periodically varying dilution rate. They did a numerical bifurcation analysis of the system and constructed its operating diagram illustrating the stable coexistence of all three populations, provided that their specific growth rate curves crossed in pairs in such a way that there existed for every population a range of values of nutrient concentration where the population had the highest specific growth rate. Thus, with periodic variation of the operating conditions, the competitive advantage alternated among the three populations. The coexistence state is usually periodic, but in the case where the specific growth rates follow Andrews's model and at the points of intersection, the slopes are opposite and the system may exhibit quasi-periodic or chaotic behavior.

Wolkowicz and Zhao [39] studied analytically the general case of n microbial species competing for a single nutrient in a periodic chemostat. They derived sufficient conditions for the coexistence of all species in a periodic state. They also pointed out an interesting feature of the system, i.e., competition mediated coexistence. This is a case where two species cannot

coexistence under certain operating conditions, but one species becomes extinct. However, simply by introducing a third species into the system under the same operating conditions extinction is avoided and all three species coexist.

It should be noted that the case of three or more populations is qualitatively different from the case of two populations. In a chemostat with two species under time-invariant operating conditions, coexistence is theoretically predicted at a semi-stable steady state and only for specific discrete values of the dilution rate where the system is structurally unstable. Periodic operation perturbs this steady state leading to a stable coexistence state and a structurally stable system. In a chemostat with more than two species, coexistence of all species in a steady state is not even theoretically possible, except for the extremely exceptional case where all the specific growth rate curves cross at the same point. For the more realistic case where the specific growth rate curves cross in pairs, only the steady states of any two of the species growing together are theoretically possible. Periodic operation leads to the coexistence of all species through the interaction of all these steady states with each other.

Periodic variation of the operating conditions of the chemostat creates temporal heterogeneity in the system, resulting that each of the microbial populations has a competitive advantage for a time interval and thus all of them coexist. Another way to achieve the coexistence of microbial populations involved in pure and simple competition is to create spatial heterogeneity. In such a system each microbial population has the competitive advantage in a different region in space and thus they coexist.

Effect of a spatially heterogeneous environment

Spatial heterogeneity may appear in a chemostat due to incomplete mixing. Spatial heterogeneity under controlled conditions is created with a configuration of interconnected chemostats, as shown in Figure 6. The equations for a two-chemostat system with pure and simple competition between two microbial populations are:

$$V_1 \frac{dx_{11}}{dt} = F_{2C}X_{12} - (F_{1E} + F_{1C})X_{11} + V_1\mu_1(S_1)X_{11} \quad (12a)$$

$$V_1 \frac{dx_{21}}{dt} = F_{2C}X_{22} - (F_{1E} + F_{1C})X_{21} + V_1\mu_2(S_1)X_{21} \quad (12b)$$

$$V_1 \frac{ds_1}{dt} = F_{1F}S_{1F} + F_{2C}S_2 - (F_{1E} + F_{1C})S_1 - V_1 \left[\frac{\mu_1(S_1)X_{11}}{Y_1} + \frac{\mu_2(S_1)X_{21}}{Y_2} \right] \quad (12c)$$

$$V_2 \frac{dx_{12}}{dt} = F_{1C}X_{11} - (F_{2E} + F_{2C})X_{12} + V_2\mu_1(S_2)X_{12} \quad (12d)$$

$$\frac{dx_{22}}{dt} = F_{1C}X_{21} - (F_{2E} + F_{2C})X_{22} + V_2\mu_2(S_2)X_{22} \quad (12e)$$

$$V_2 \frac{ds_2}{dt} = F_{2F}S_{2F} + F_{1C}S_1 - (F_{2E} + F_{2C})S_2 - V_2 \left[\frac{\mu_1(S_2)X_{11}}{Y_1} + \frac{\mu_2(S_2)X_{21}}{Y_2} \right] \quad (12f)$$

The first three equations are the balances for the two microbial populations and the rate-limiting nutrient in the first chemostat and the next three equations are the corresponding balances in the second chemostat.

The system of eq. (12) was studied by Stephanopoulos and Fredrickson [40] with Monod's model describing the specific growth rates of the populations. They showed that the coexistence of the two populations is obtained in a stable steady state. Coexistence is achieved because the conditions in each chemostat favor a different population. This, of course, requires that the specific growth rate curves cross, so that each population has the highest specific growth rate for a range of nutrient concentrations. Then, at the nutrient concentration in the first chemostat one population grows faster, but at the nutrient concentration in the second chemostat the other population grows faster. Kung and Baltzis [41] studied the effect of the operating parameters, and Jäger et al. [42], Smith and Tang [43] and Hofbauer and So [44] derived analytic conditions for coexistence. Smith and Waltman [45] showed that the coexistence steady state is unique and globally stable. The more general case of the competition of two microbial populations in a n -chemostat configuration has been also studied [44-46]. It has been concluded that coexistence is more likely to occur as the number of vessels increases [46]. Also, it was found that, in the case of a three-chemostat configuration, coexistence may occur in multiple steady states. Lenas et al. [47] studied the system of equations (12) when the specific growth rates were described by Andrews's model and in particular when the specific growth rate curves crossed with opposite slopes. They found that coexistence was obtained, not only in a steady state, but also in a periodic state. This is a counterintuitive result, since the system of one chemostat [eq. (1)] does not exhibit oscillations. It appears that oscillations are generated through the coupling of the chemostats from the perturbation of their steady state with double-zero eigenvalue. As mentioned already, the double-zero eigenvalue is probably responsible for quasi-periodicity and chaos in a periodically operated chemostat. Also, it is probably responsible for the oscillations that arise when the feed in the system of the single chemostat is not sterile, but contains two species [48].

For the general case of pure and simple competition of n microbial populations one would expect that a configuration of n interconnected chemostat is required for their coexistence, since for each population a chemostat must exist where the

population has the competitive advantage over the other populations. Jäger et al. [49] showed that no more than n population competing for a single rate-limiting nutrient can coexist in a configuration of n chemostats. El-Owaidy et al. [50] showed that a necessary condition for the coexistence of three populations in a configuration of three chemostats is the three steady states of coexistence of the populations in pairs to exist simultaneously. However, extensive computations by Chang and Baltzis [51] failed to determine regions of the operating parameter space in which stable coexistence can be realized. Coexistence was found only for discrete values of the operating parameters where the system was structurally unstable. The coexistence of three competing microbial populations was shown to be possible in a stable steady state in configurations of four interconnected chemostats [52]. In the case where the specific growth rate curves are described by Andrews's model and oscillations are generated in a configuration of interconnected chemostats [47], one may expect that a number of populations exceeding the number of reactors could coexist, not in a steady state, but in a periodic state. However, no such study exists at present.

The effect of spatial heterogeneity on microbial competition can be also studied in a stagnant film in which the microorganisms grow and through which the rate-limiting nutrient diffuses. The nutrient enters from one end of the film, diffuses through it and is consumed by the microorganisms (Figure 7). Kelly et al. [53] studied the competition between two microbial species for one rate-limiting nutrient in this system. In such an environment the motility of the cells of the two species plays an important role in the outcome of the competition. Besides random motility, chemotactic motion of the cells towards higher nutrient concentrations was considered. The equations describing the system are:

$$\frac{\partial x_1}{\partial t} = m_1 \frac{\partial^2 x_1}{\partial z^2} - \delta_1 \frac{\partial}{\partial z} \left[\frac{1}{(K_1+s)^2} \frac{\partial s}{\partial z} x_1 \right] + \mu_1(s) x_1 \quad (13a)$$

$$\frac{\partial x_2}{\partial t} = m_2 \frac{\partial^2 x_2}{\partial z^2} - \delta_2 \frac{\partial}{\partial z} \left[\frac{1}{(K_2+s)^2} \frac{\partial s}{\partial z} x_2 \right] + \mu_2(s) x_2 \quad (13b)$$

$$\frac{\partial s}{\partial t} = D_s \frac{\partial^2 s}{\partial z^2} - \frac{1}{Y_1} \mu_1(s) x_1 - \frac{1}{Y_2} \mu_2(s) x_2 \quad (13c)$$

The first term on the right-hand side of eq. (13a)–(13b) represents the random motility of the cells and the second term chemotactic motion. The coefficients m_1 and m_2 denote the magnitude of random motility and the coefficients δ_1 and δ_2 represent the intensity of chemotaxis. The boundary conditions accompanying eq. (13) are:

$$z = 0: \frac{\partial x_1}{\partial z} = \frac{\partial x_2}{\partial z} = \frac{\partial s}{\partial z} = 0 \quad (14a)$$

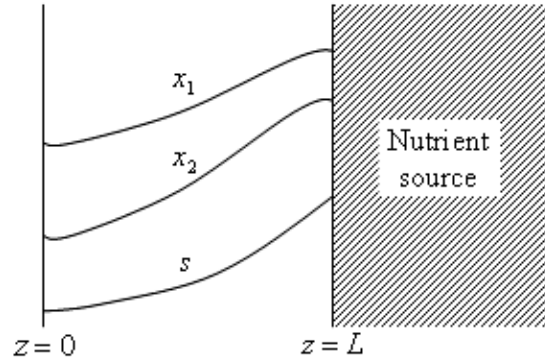


Figure 7. Model of microbial competition in conditions of spatial heterogeneity

$$z = L: \frac{\partial x_1}{\partial z} = \frac{\partial x_2}{\partial z} = 0, \quad s = s_0 \quad (14b)$$

The specific growth rates were assumed to follow Monod's model with population 1 having the highest specific growth rate for all nutrient concentrations (case in Figure 1a), *i.e.*, population 1 has the advantage with respect to growth. The interest lies in whether differences in the motility characteristics can give the competitive advantage to the slower growing population 2. It was shown that this is indeed the case. A lower random motility coefficient or a higher chemotactic coefficient enable population 2 to coexist with or even outcompete population 1.

A way to model imperfect mixing in a bioreactor is by considering a tubular reactor with backmixing. Such a model was studied by Kung and Baltzis [54] and its equations are:

$$\frac{\partial x_1}{\partial t} = D \frac{\partial^2 x_1}{\partial z^2} - u \frac{\partial x_1}{\partial z} + \mu_1(s) x_1 \quad (15a)$$

$$\frac{\partial x_2}{\partial t} = D \frac{\partial^2 x_2}{\partial z^2} - u \frac{\partial x_2}{\partial z} + \mu_2(s) x_2 \quad (15b)$$

$$\frac{\partial s}{\partial t} = D \frac{\partial^2 s}{\partial z^2} - u \frac{\partial s}{\partial z} - \frac{1}{Y_1} \mu_1(s) x_1 - \frac{1}{Y_2} \mu_2(s) x_2 \quad (15c)$$

with the boundary conditions:

$$z = 0: u x_1 - D \frac{\partial x_1}{\partial z} = 0, \quad u x_2 - D \frac{\partial x_2}{\partial z} = 0, \\ u s - D \frac{\partial s}{\partial z} = u s_f \quad (16a)$$

$$z = L: \frac{\partial x_1}{\partial z} = \frac{\partial x_2}{\partial z} = \frac{\partial s}{\partial z} = 0 \quad (16b)$$

The dispersion coefficient D is a measure of the degree of mixing and u is the flow velocity through the reactor. At the limit $D = 0$ we obtain an ideal plug-flow tubular reactor and at the limit $D \rightarrow \infty$ we have perfect mixing as in an ideal chemostat. Monod's model was considered for specific growth rates and stable

coexistence was found when the specific growth rate curves crossed (case in Figure 1b). The operating diagram of the system is similar to the one shown in Figure 5, with D_0 replaced by u .

Even in a perfectly mixed chemostat, spatial heterogeneity can be created from attachment of the cells on the chemostat walls. Baltzis and Fredrickson [55] studied the case of pure and simple competition between two microbial populations in a chemostat when one of them has the ability to attach to the chemostat walls. For the attachment process they used two different models: the Topiwala and Hamer model [56], which assumes irreversible attachment to the surface, and its generalization, the model of reversible attachment. For the Topiwala and Hamer model the equations of the system are:

$$\frac{dx_1}{dt} = -Dx_1 + \mu_1(s) x_1 \quad (17a)$$

$$\frac{dx_2}{dt} = -Dx_2 + \mu_2(s) \left(x_2 + \frac{A}{V} \dot{x}_2 \right) \quad (17b)$$

$$\frac{ds}{dt} = D(s_F - s) - \frac{1}{Y_1} \mu_1(s) x_1 - \frac{1}{Y_2} \mu_2(s) \left(x_2 + \frac{A}{V} \dot{x}_2 \right) \quad (17c)$$

and for the model of reversible attachment:

$$\frac{dx_1}{dt} = -Dx_1 + \mu_1(s) x_1 \quad (18a)$$

$$\frac{dx_2}{dt} = -Dx_2 + \mu_2(s) \left(x_2 + \frac{A}{V} \dot{x}_2 \right) - \frac{A}{V} \left[kx_2(x_{2m} - x_2) - k' \dot{x}_2 \right] \quad (18b)$$

$$\frac{d\dot{x}_2}{dt} = kx_2(x_{2m} - x_2) - k' \dot{x}_2 \quad (18c)$$

$$\frac{ds}{dt} = D(s_F - s) - \frac{1}{Y_1} \mu_1(s) x_1 - \frac{1}{Y_2} \mu_2(s) \left(x_2 + \frac{A}{V} \dot{x}_2 \right) \quad (18d)$$

In the above equations, only population 2 is assumed to have the ability for wall attachment. The cell surface density \dot{x}_2 is constant in the Topiwala and Hamer model, but variable in the reversible attachment model. The second model considers both the processes of attachment and detachment. Attachment of the members of population 2 on the chemostat walls gives an advantage to it over population 1, since it makes its washout of the chemostat impossible in the first model and more difficult in the second one. Thus, if population 1 has the advantage with respect to the magnitude of the specific growth rate, coexistence of the two species is possible. It is not necessary for the specific growth rate curves to cross for coexistence (case in Figure 1a),

as long as the population with the ability of attachment has also the lower specific growth rate.

All the studies of pure and simple competition are mainly concerned with the coexistence of the involved species and under which conditions it is possible. When competition is for more than one nutrient, again the main question is about the survival and the coexistence of the species.

COMPETITION FOR TWO OR MORE NUTRIENTS

In general, when we have the total competition of n microbial populations for m nutrients in a chemostat with $n > m$, no more than m populations can grow in the chemostat at steady state. This can be shown by analyzing the equations of the chemostat:

$$\frac{dx_i}{dt} = Dx_i + \mu_i(s_1, \dots, s_m) x_i \quad i=1, \dots, n \quad (19a)$$

$$\frac{ds_j}{dt} = D(s_{jF} - s_j) - \sum_{i=1}^n r_{ij}(s_1, \dots, s_m) x_i \quad j=1, \dots, m \quad (19b)$$

The first equation contains the balances for the n microbial species and the second equation contains the balances for the m nutrients. The specific growth rates μ_i and the specific feeding rates r_{ij} are, in general, functions of the concentrations of the m nutrients. At a steady state where all the species are present ($x_i > 0$), eq. (19a) yields:

$$D = \mu_i(s_1, \dots, s_m) \quad i=1, \dots, n \quad (20)$$

This is a system of n non-linear algebraic equations with m unknowns, and, in general, a necessary condition for a solution is that $n \leq m$. This means that no more than m species can coexist on m nutrients. If we take the dilution rate D to also be an unknown, then there may also exist a solution when $n = m + 1$, but then the dilution rate must be fixed at a specific value and the system there is structurally unstable, as in the case of two populations competing for one nutrient. It should be emphasized that the condition just stated is a necessary one and does not ensure the coexistence of the species.

Modeling studies [57–64] of two microbial populations competing for two rate-limiting nutrients ($n = m = 2$) show that stable coexistence can be obtained for a range of operating conditions. Butler and Wolkovicz [63] have also shown that, in the case that the nutrients are inhibitory, there can also be periodic coexistence. Stable coexistence has also been demonstrated experimentally [59,62]. Furthermore, Gottschal and Thingstad [62] have studied theoretically and experimentally the case of three microbial populations competing for two rate-limiting nutrients in a chemostat ($n = 3, m = 2$). Their conclusion was that the coexistence of all three populations was not possible, but up to two populations could survive, while the third one became extinct. Li and Smith [65] have shown that

all solutions of the model equations for this case converge either to a state of one–species survival or of two–species survival. The case of three microbial populations competing for three rate–limiting nutrients in a chemostat ($n=3$, $m=3$) has been studied by Vayenas and Pavlou [66], by Husiman and Weissing [67], and by Li [68] for different expressions for the specific growth rates. In all cases stable coexistence was found, not only in a steady state, but also in a periodic state. This observation brings forth the expectation that periodic oscillations generated in the system would allow the coexistence of more species in a periodic state. Indeed, Li and Smith [69] have shown the existence of a periodic solution in which four species coexist in competition for three resources and noted that their arguments could be extended to more species. Husiman and Weissing [67] also showed the generation of oscillations in a system of three species competing for three resources and demonstrated that chaotic oscillations occur when five species are involved. They concluded that up to nine species could be supported by three nutrients.

Competition for more than one nutrient has been also studied in configurations of interconnected chemostats [70,71]. Thomopoulos et al. [71] studied the competition of three species for two rate–limiting nutrients in configurations of two and three chemostats. In the two–chemostat case the coexistence of all three species is possible only at discrete values of the operating parameters where the system is structurally unstable. However, in the three–chemostat case coexistence in a stable steady state was observed.

CONCLUSIONS

Microbial competition, being one of the most common interactions among microbial species inhabiting the same environment, has been the subject of numerous analytic, computational and experimental studies. The main question in those studies is how many of the competing species and under which conditions can coexist. Periodic variation of the environmental conditions, modeled in a chemostat by periodic variation of one of the operating parameters, offers a means of obtaining stable coexistence in the form of sustained periodic, and sometimes, quasi–periodic or chaotic oscillations. The spatial heterogeneity of the environment allows the competing species to coexist in a stable steady state. Spatial heterogeneity is often modeled by a configuration of interconnected chemostats, each chemostat representing a sub–environment where one of the species has a competitive advantage over the others. Coexistence is obtained by allowing each of the species to have a competitive advantage in one of the chemostats. The question of coexistence in a stable steady state of n microbial populations competing for m nutrients in a configuration of k interconnected chemostats is a general one and has been answered only in special

Table 1. Coexistence in a steady state of n microbial populations competing for m nutrients in a configuration of k interconnected chemostats

Number of species (n)	Number of nutrients (m)	Number of chemostats (k)	Coexistence
2	1	1	in a structurally unstable system
2	1	2	stable
2	2	1	stable
3	1	3	in a structurally unstable system
3	1	4	stable
3	2	2	in a structurally unstable system
3	2	3	stable
3	3	1	stable

cases for n , m and k values. These cases are summarized in Table 1. One expects coexistence to be facilitated as the number of nutrients m or the number of reactors k increases, or the number of competing species n decreases. The cases listed in the table where coexistence is observed in a structurally unstable system are limiting ones. Increasing m or k or decreasing n results in coexistence in a stable steady state. On the other hand, decreasing m or k or increasing n makes coexistence in a steady state impossible. However, periodic oscillations generated in the system can in some cases increase the number of coexisting species, but in a stable periodic case and not in a steady state.

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REFERENCES

- [1] A.G. Fredrickson, G. Stephanopoulos, Microbial competition, *Science* **213** (1981) 972–979.
- [2] E.O. Powell, Criteria for growth of contaminants and mutants in continuous culture, *J. Gen. Microbiol.* **18** (1958) 259–268.
- [3] J. Monod, *Recherches sur la croissance des culture bactériennes*, Hermann Et Cie., Paris 1942.
- [4] A.G. Fredrickson, H.M. Tsuchiya, in *Chemical reactor theory: a review*, N.R. Amundson, L. Lapidus, Eds., Prentice–Hall, Englewood Cliffs, Nj 1977, p. 405.
- [5] R. Aris, A. E. Humphrey, Dynamics of a chemostat in which two organisms compete for a common substrate, *Biotechnol. Bioeng.* **19** (1977) 1375–1386.
- [6] G.E. Powell, Structural instability of the theory of simple competition, *J. Theor. Biol.* **132** (1988) 421–435.
- [7] J.F. Andrews, A mathematical model for the continuous culture of microorganisms utilizing inhibitory substrates, *Biotechnol. Bioeng.* **10** (1968) 707–723.
- [8] P. Lenas, S. Pavlou, Periodic, quasi–periodic and chaotic coexistence of two competing microbial population in a

- periodically operated chemostat, *Math. Biosci.* **121** (1994) 61–110.
- [9] G.J. Butler, G.S.K. Wolkowicz, A mathematical model of the chemostat with a general class of functions describing nutrient uptake, *SIAM J. Appl. Math.* **45** (1985) 138–151.
- [10] G.S.K. Wolkowicz, L. Zhiqi, Global dynamics of a mathematical model of competition in the chemostat: general response functions and differential death rates, *SIAM J. Appl. Math.* **52** (1992) 222–233.
- [11] B. Li, Global asymptotic behavior of the chemostat: general response functions and different removal rates, *SIAM J. Appl. Math.* **59** (1999) 411–422.
- [12] S.B. Hsu, S. Hubbell, P. Waltman, A mathematical theory for single-nutrient competition in continuous cultures of micro-organisms, *SIAM J. Appl. Math.* **32** (1977) 366–383.
- [13] S.B. Hsu, Limiting behaviour for competing species, *SIAM J. Appl. Math.* **34** (1978) 760–763.
- [14] G. Stephanopoulos, R. Aris, A.G. Fredrickson, A stochastic analysis of the growth of competing microbial populations in a continuous biochemical reactor, *Math. Biosci.* **45** (1979) 99–135.
- [15] D.W. Tempest, J.W. Dicke, J.L. Meers, Magnesium-limited growth of *Bacillus subtilis*, in pure and mixed culture, in a chemostat, *J. Gen. Microbiol.* **49** (1967) 139–147.
- [16] H.W. Jannasch, Enrichments of aquatic bacteria in continuous culture, *Arch. Mikrobiol.* **59** (1967) 165–173.
- [17] H.W. Jannasch, Competitive eliminations of Enterobacteriaceae from seawater, *Appl. Microbiol.* **16** (1968) 1616–1618.
- [18] J.L. Meers, D.W. Tempest, The influence of extracellular products on the behaviour of mixed microbial populations in magnesium-limited chemostat cultures, *J. Gen. Microbiol.* **52** (1968) 309–317.
- [19] W. Harder, H. Veldkamp, Competition of marine psychrophilic bacteria at low temperatures, *Antonie van Leeuwenhoek J. Microbiol. Serol.* **37** (1971) 51–63.
- [20] J.L. Meers, Effect of dilution rate on the outcome of chemostat mixed culture experiments, *J. Gen. Microbiol.* **67** (1971) 359–361.
- [21] R.D. Megee III, J.F. Drake, A.G. Fredrickson, H.M. Tsuchiya, Studies in intermicrobial symbiosis. *Saccharomyces cerevisiae* and *Lactobacillus casei*, *Can. J. Microbiol.* **18** (1972) 1733–1742.
- [22] J.L. Jost, J.F. Drake, A.G. Fredrickson, H.M. Tsuchiya, Interactions of *Tetrahymena pyriformis*, *Escherichia coli*, *Azotobacter vinelandii*, and glucose in a minimal medium, *J. Bacteriol.* **113** (1973) 834–840.
- [23] S.R. Hansen, S.P. Hubbell, Single-nutrient microbial competition: qualitative agreement between experimental and theoretically forecast outcomes, *Science* **207** (1980) 1491–1493.
- [24] K. Takeya, A. Kuwata, M. Yoshida, T. Miyazaki, Effect of dilution rate on competitive interactions between the cyanobacterium *Microcystis novacekii* and the green alga *Scenedesmus quadricauda* in mixed chemostat cultures, *J. Plankton Res.* **26** (2004) 29–35.
- [25] G. Hardin, The competitive exclusion principle, *Science* **131** (1960) 1292–1297.
- [26] G.E. Hutchinson, The paradox of the plankton, *Am. Nat.* **95** (1961) 137–144.
- [27] G. Stephanopoulos, A.G. Fredrickson, R. Aris, The growth of competing microbial populations in a CSTR with periodically varying inputs, *AIChE J.* **25** (1979) 863–872.
- [28] F.M. Stewart, B.R. Levin, Partitioning of resources and the outcome of interspecific competition: a model and some general considerations, *Am. Nat.* **107** (1973) 171–198.
- [29] M. Matsubara, N. Watanabe, S. Hasegawa, Bifurcations in a bang-bang controlled mixed culture system, *Chem. Eng. Sci.* **41** (1986) 523–531.
- [30] G.J. Butler, S.B. Hsu, P. Waltman, A mathematical model for the chemostat with periodic washout rate, *SIAM J. Appl. Math.* **45** (1985) 435–449.
- [31] S.B. Hsu, A competition model for a seasonally fluctuating nutrient, *J. Math. Biol.* **9** (1980) 115–132.
- [32] H. Smith, Competitive coexistence in an oscillating chemostat, *SIAM J. Appl. Math.* **40** (1981) 498–522.
- [33] J.K. Hale, A.S. Somolinos, Competition for fluctuating nutrient, *J. Math. Biol.* **18** (1983) 255–280.
- [34] B.H. Davison, G. Stephanopoulos, Effect of pH oscillations on a competing mixed culture, *Biotechnol. Bioeng.* **28** (1986) 1127–1137.
- [35] S. Dikshitulu, B.C. Baltzis, G.A. Lewandowski, S. Pavlou, Competition between two microbial populations in a sequencing fed-batch reactor: theory, experimental verification, and implications for waste treatment applications, *Biotechnol. Bioeng.* **42** (1993) 643–656.
- [36] M.L. Stephens, G. Lyberatos, Effect of cycling on final mixed culture fate, *Biotechnol. Bioeng.* **29** (1987) 672–678.
- [37] S. Pavlou, I.G. Kevrekidis, G. Lyberatos, On the coexistence of competing microbial species in a chemostat under cycling, *Biotechnol. Bioeng.* **35** (1990) 224–232.
- [38] P. Lenas, S. Pavlou, Coexistence of three competing microbial populations in a chemostat with periodically varying dilution rate, *Math. Biosci.* **129** (1995) 111–142.
- [39] G.S.K. Wolkowicz, X.Q. Zhao, n-species competition in a periodic chemostat, *Differential & Integral Equations* **11** (1998) 465–491.
- [40] G. Stephanopoulos, A.G. Fredrickson, Effect of spatial inhomogeneities on the coexistence of competing microbial populations, *Biotechnol. Bioeng.* **21** (1979) 1491–1498.
- [41] C.M. Kung, B.C. Baltzis, Operating parameters' effects on the outcome of pure and simple competition between two populations in configurations of two interconnected chemostats, *Biotechnol. Bioeng.* **30** (1987) 1006–1018.
- [42] W. Jäger, J.W.H. So, B. Tang, P. Waltman, Competition in the gradostat, *J. Math. Biol.* **25** (1987) 23–42.
- [43] H. Smith, B. Tang, Competition in the gradostat: the role of the communication rate, *J. Math. Biol.* **27** (1989) 139–165.
- [44] J. Hofbauer, J.W.H. So, Competition in the gradostat: the global stability problem, *Nonlin. Anal., Theory, Methods Applic.* **22** (1994) 1017–1031.
- [45] H.L. Smith, P. Waltman, The Gradostat: A model of competition along a nutrient gradient, *Microb. Ecol.* **22** (1991) 207–226.
- [46] H.L. Smith, B. Tang, P. Waltman, Competition in an n-vessel gradostat, *SIAM J. Appl. Math.* **51** (1991) 1451–1471.
- [47] P. Lenas, N.A. Thomopoulos, D.V. Vayenas, S. Pavlou, Oscillations of two competing microbial populations in configurations of two interconnected chemostats, *Math. Biosci.* **148** (1998) 43–63.
- [48] A. Ajbar, K. Alhumazi, Microbial competition: study of global branching phenomena, *AIChE J.* **46** (2000) 321–334.
- [49] W. Jäger, H. Smith, B. Tang, in *Differential equations. Models in Biology, Epidemiology and Ecology: Proceedings, Claremont 1990*, S. Busenberg, M. Martelli, Eds., Springer-Verlag, New York, 1992, p. 200.
- [50] H. El-Owaidy, A.A. Ammar, O.A. El-Leithy, Competition in the gradostat for the case of three vessels and three populations, *Tamkang J. Math.* **21** (1990) 143–151.

- [51] S.W. Chang, B.C. Baltzis, Impossibility of coexistence of three pure and simple competitors in configurations of three interconnected chemostats, *Biotechnol. Bioeng.* **33** (1989) 460–470.
- [52] B.C. Baltzis, M. Wu, Steady-state coexistence of three pure and simple competitors in a four-membered reactor network, *Math. Biosci.* **123** (1994) 147–165.
- [53] F.X. Kelly, K.J. Dapsis, D.A. Lauffenburger, Effect of bacterial chemotaxis on dynamics of microbial competition, *Microbial Ecol.* **16** (1988) 115–131.
- [54] C.M. Kung, B.C. Baltzis, The growth of pure and simple microbial competitors in a moving distributed medium, *Math. Biosci.* **111** (1992) 295–313.
- [55] B.C. Baltzis, A.G. Fredrickson, Competition of two microbial populations for a single resource in a chemostat when one of them exhibits wall attachment, *Biotechnol. Bioeng.* **25** (1983) 2419–2439.
- [56] H.H. Topiwala, G. Hamer, Effect of wall growth in steady-state continuous culture, *Biotechnol. Bioeng.* **13** (1971) 919–922.
- [57] P.A. Taylor, P.J.L. Williams, "Theoretical studies on the coexistence of competing species under continuous-flow conditions, *Can. J. Microbiol.* **21** (1975) 90–98.
- [58] J.A. Léon, D.B. Tumpson, Competition between two species for two complementary or substitutable resources, *J. Theor. Biol.* **50** (1975) 185–201.
- [59] H. Yoon, G. Klinzing, H.W. Blanch, Competition for mixed substrates by microbial populations, *Biotechnol. Bioeng.* **19** (1977) 1193–1210.
- [60] S. Gondo, K. Kaushik, K. Venkatasubramanian, Two-(carbon)-substrate continuous cultures: multiple steady states and their stability, *Biotechnol. Bioeng.* **20** (1978) 1479–1485.
- [61] S.B. Hsu, K.S. Cheng, S.P. Hubbell, Exploitative competition of microorganisms for two complementary nutrients in continuous cultures, *SIAM J. Appl. Math.* **41** (1981) 422–444.
- [62] J.C. Gottschal, T.F. Thingstad, "Mathematical description of competition between two and three bacterial species under dual substrate limitation in the chemostat: a comparison with experimental data", *Biotechnol. Bioeng.* **24** (1982) 1403–1418.
- [63] G.J. Butler, G.S.K. Wolkowicz, Exploitative competition in a chemostat for two complementary, and possibly inhibitory, resources, *Math. Biosci.* **83** (1987) 1–48.
- [64] M.M. Ballyk, G.S.K. Wolkowicz, Exploitative competition in the chemostat for two perfectly substitutable resources, *Math. Biosci.* **118** (1993) 127–180.
- [65] B.T. Li, H. Smith, How many species can two essential resources support?, *SIAM J. Appl. Math.* **62** (2001) 336–366.
- [66] D.V. Vayenas, S. Pavlou, Coexistence of three microbial populations competing for three complementary nutrients in a chemostat, *Math. Biosci.* **161** (1999) 1–13.
- [67] J. Huisman, F.J. Weissing, Biodiversity of plankton by species oscillations and chaos, *Nature* **402** (1999) 407–410.
- [68] B.T. Li, Periodic coexistence in the chemostat with three species competing for three essential resources, *Math. Biosci.* **174** (2001) 27–40.
- [69] B.T. Li, H.L. Smith, Periodic coexistence of four species competing for three essential resources, *Math. Biosci.* **184** (2003) 115–135.
- [70] A.A.S. Zaghrout, Asymptotic behaviour of solutions of competition in gradostat with two limiting complementary substrates, *Appl. Math. and Comp.* **49** (1992) 19–37.
- [71] N.A. Thomopoulos, D.V. Vayenas and S. Pavlou, On the coexistence of three microbial populations competing for two complementary substrates in configurations of interconnected chemostats, *Math. Biosci.* **154** (1998) 87–102.

IZVOD

KOMPETICIJA MIKROORGANIZAMA U BIOREKTORIMA

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Priliko razvoja više različitih mikroorganizama u određenoj sredini česta je pojava kompeticije u pogledu korišćenja hranljivih materija i drugih nutrijenata. Ovakva sredina se može kreirati u laboratorijskom bioreaktoru u cilju proučavanja ovakvog tipa interakcije i njegovog efekta na mikroorganizme. Kompeticija ima tendenciju da eliminiše mikroorganizme iz sistema. Osnovno pitanje koje se ovde postavlja je da li kompeticija između više mikroorganizama može da dovede i pod kojim uslovima do njihove koegzistencije. Kompleksnost (broj) hranljivih materija koje predstavljaju hranu za različite mikroorganizme kao uzrok kompeticije važan je parametar, pri čemu je utvrđeno da periodične oscilacije i specifična heterogenost ima povoljan efekat na postojanje koegzistencije između mikroorganizama.

Ključne reči: Bioreaktor, Microbna fermentacija, Mešane kulture, Kompeticija.